ABSTRACT. We show that for any convex body $K \subset E^2$ there exists a triangle $T$ such that $T \subset K \subset (3(\sqrt{17} - 1)/4)T$, where $\lambda T$ is a suitable homothetic copy of $T$ with ratio $\lambda$. As a corollary we show that if $(K_i)$ are homothetic copies of a given convex body $K \subset E^2$ with area $V(K) = 1$, then the condition $\Sigma V(K_i) \geq 9(9 - \sqrt{17})/4$ is sufficient for the existence of a translative covering of $K$ by $(K_i)$.

We define a convex body as a compact convex subset of Euclidean $n$-space $E^n$ with interior points. The class of all convex bodies of $E^n$ will be denoted by $C^n$. Let $K \in C^n$ and $(K_i)$ a finite or infinite sequence of convex bodies of $C^n$. We say that $(K_i)$ permits a translative covering of $K$ if there are translations $\tau_i$ so that $(\tau_i K_i)$ is a covering of $K$, i.e. $K \subset \bigcup \tau_i K_i$. The volume of a set $M \subset E^n$ will be denoted by $V(M)$.

The following problem has been formulated in [9, Prob. 111]. Let $K \in C^n$ be such that $V(K) = 1$. What is the least positive number $f_2(K)$ such that any sequence $(K_i)$ of homothetic copies of $K$ permits a translative covering of $K$ as soon as $\Sigma V(K_i) \geq f_2(K)$. It should be remarked that we consider only positive homothetic ratios in this paper.

For the unit hypercube $S$ in $E^n$ it is known that

$$f_n(S) = 2^n - 1$$

(see Meir and Moser [7]), and in view of invariance of $f_n(K)$ with regard to affine transformations of $E^n$ it follows that

$$f_n(\mathcal{P}) = 2^n - 1$$

for any $n$-dimensional parallelepiped $\mathcal{P}$ of unit volume.

Further, A. Bezdek and Z. Füredi proved the following unpublished result:

$$f_2(T) = 2$$

for any triangle $T$ of unit area.

L. Fejes-Tóth has conjectured that $2 \leq f_2(K) \leq 3$ for any planar convex region of unit area, but the only known result in this direction is

$$f_2(K) \leq 12$$

From Bezdek and Bezdek [1].
An improvement of (4) can be obtained as a corollary of the following theorem, which is the main result of this paper and is also of independent interest.

**THEOREM.** For every convex body \( K \subset \mathbb{E}^2 \) there are a triangle \( T_1 \) and an image \( T_2 \) of \( T_1 \) under a homothety with ratio \( \frac{3(\sqrt{17} - 1)}{4} \) such that \( T_1 \subset K \subset T_2 \).

**Proof.** Let \( K \subset \mathbb{E}^2 \) be a convex body. Let \( T_1, T_2 \) be homothetic triangles such that \( T_1 \subset K \subset T_2 \). Let us denote by \( k(T_1, T_2) \) the ratio of the homothety which maps \( T_1 \) onto \( T_2 \). We call the number

\[
T_1(K) = \min\{k(T_1, T_2); T_1 \subset K \subset T_2\}
\]

the triangular index of \( K \). Now it is sufficient to prove the inequality

\[
T_1(K) \leq \frac{3(\sqrt{17} - 1)}{4}.
\]

Let \( T_1 \), with vertices \( A, B, C \), be a triangle of maximal area such that \( T_1 \subset K \). The existence of such a \( T_1 \) results from the compactness of \( K \). Let \( T_2 \), with vertices \( A', B', C' \), be a triangle of minimal area homothetic with \( T_1 \) such that \( K \subset T_2 \). Let us choose the unit of length such that the area \( P(ABC) \) of the triangle \( ABC \) satisfies

\[
P(ABC) = 1.
\]

We draw through the points \( A, B, C \) the straight lines \( p_1 \parallel BC, p_2 \parallel CA, p_3 \parallel AB \), respectively. The intersection \( B'C' \cap K \) is non-empty, since the triangle \( A'B'C' \) has minimal area. Consequently, the distance from the line \( B'C' \) to \( BC \) is less than or equal to the distance from \( A \) to \( BC \). Analogously for the lines \( A'B', A'C' \). The intersection of the half-planes \( p_1 B, p_2 A, p_3 C \) and of the triangle \( A'B'C' \), is, in general, a hexagon; we denote its vertices by \( V_1, V_2, \ldots, V_6 \) (see Figure 1).

Further, we denote the vectors \( \overrightarrow{AB} \) by \( u \) and \( \overrightarrow{AC} \) by \( v \). Then

\[
V_1 = A + u + \alpha v, \quad \text{where } 0 \leq \alpha \leq 1,
\]
\[
V_2 = A + v + \alpha u,
\]
\[
V_3 = A + v - \beta u, \quad \text{where } 0 \leq \beta \leq 1,
\]
\[
V_4 = A - \beta(u - v)
\]
\[
V_5 = A + \gamma(u - v), \quad \text{where } 0 \leq \gamma \leq 1,
\]
\[
V_6 = A + u - \gamma v.
\]