Basis Functions for the Buckling of Plates

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Summary

We form basis functions on an angular region for the buckling of plates. Convergence of these functions is established under mild hypotheses. The basis functions are useful in the study of buckling of any polygonal plate.

Introduction

In this paper, we form basis functions for the equation

$$\nabla^4 w = \left( \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial \theta} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} \right)^2 w = \frac{N}{D} \nabla^2 w$$

(1)

on an infinite angular region. Eq. (1) is the deflection equation of a plate when uniform forces per unit length are applied in the middle plane of the plate along both $x$ and $y$ directions.

Such basis functions have application to plate problems ([2], [3]). For example, a quadrilateral plate can be divided by a diagonal into two finite angular regions. On each angular region, the solution will be a linear combination of the basis functions. The two solutions can be matched along the diagonal to find the coefficients of the basis functions [2].

Let the angular region $AOB$ be such that $OA$ is along the positive $x$-axis, and $\theta_s = \angle BOA$ where $0^\circ < \theta_s < 180^\circ$. The edges $OA$ and $OB$ can have any of the classical homogeneous boundary conditions. For example, if the edges are clamped, the boundary conditions are given by

$$w(r, 0) = w(r, \theta_s) = 0$$

$$\frac{\partial w}{\partial \theta} (r, 0) = \frac{\partial w}{\partial \theta} (r, \theta_s) = 0.$$  

(2)

We note that each basis function satisfies the differential equation and the boundary conditions exactly on the angular region $AOB$. These also have
been known in the literature as eigenfunctions ([2], [5]). We call them basis functions since the solution on an angular region is a linear combination of these functions.

Such basis functions have been used to get the deflection of a polygonal plate under loading ([2], [5]). In [3], these are used to find the frequency and mode shape of vibration of quadrilateral plates.

The superiority of the method is in its applicability to any polygonal plate under any of the classical boundary conditions. For an account of the application of the continuity conditions, see the references given above. The motivation for developing the basis functions is as follows. No theoretically exact solution on an angular region which satisfies both the differential equation and arbitrary boundary conditions exists. Theoretically exact solutions are available only for simply supported boundary conditions. Even for rectangular plates, no exact solution is available under arbitrary boundary conditions. The basis functions introduced here are theoretically exact on an angular region under arbitrary boundary conditions. Once these basis functions are formed, a collocation procedure could be used to get the critical load or mode shape of deflection of any quadrilateral plate.

**Basis Functions**

The method of solution of (1) and (2) can be indicated as follows. Let \( w_0(r, \Theta) \) be such that \( V^4 w_0 = 0 \), and \( w_1(r, \Theta) \) be such that \( V^4 w_1 = \mu V^2 w_0 \), where \( \mu = N/D \). Continuing in this fashion, we define \( w_{i+1} \) by \( V^4 w_{i+1} = \mu V^2 w_i \). Suppose \( w_i, i = 0, 1, 2, \ldots \) satisfy the given boundary conditions. Then \( w = \sum_{i=0}^{\infty} w_i \) solves \( V^4 w = \mu V^2 w \) and the boundary conditions.

Choosing

\[
 w_0(r, \Theta) = r^{\lambda+1}(A_0 \cos (\lambda + 1) \Theta + B_0 \sin (\lambda + 1) \Theta + C_0 \cos (\lambda - 1) \Theta + D_0 \sin (\lambda - 1) \Theta), \tag{3}
\]

the boundary conditions (2) for the clamped case imply that the determinant

\[
\begin{vmatrix}
1 & 0 \\
\cos (\lambda + 1) \Theta & \sin (\lambda + 1) \Theta \\
0 & 1 \\
-(\lambda + 1) \sin (\lambda + 1) \Theta & (\lambda + 1) \cos (\lambda + 1) \Theta \\
\end{vmatrix} = 0. \tag{4}
\]