DETERMINING THE TORQUE TRANSMITTED BY A
"DRY-FLUID" COUPLING

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Formulas are derived for calculating the torque transmitted by an electromagnetic "dry-fluid" coupling as a function of the rheological characteristics of the working medium as well as the size and the shape of the active surfaces.

The magnetorheological properties of ferromagnetic dispersions have been utilized in the development of very efficient "dry-fluid" couplings, which offer several advantages over other types. Such couplings are becoming more and more widespread in engineering and industrial practice, typically in metal processing [1]. As yet, however, there is no known method by which the dynamic performance of such couplings can be calculated taking into account the magnetorheological properties of the working medium.

The authors attempt here to determine, to the first approximation, the torque transmitted by a "dry-fluid" coupling with a medium the rheological properties of which in a magnetic field are described by the following defining equation:

\[ p_{ik} = -\delta_{ik} \rho + 2k\eta_0 - \epsilon_{ik}. \]  \hspace{1cm} (1)

It is well known that, in the phenomenological sense, the rheological properties of many dispersion systems are adequately well described by this equation [2, 3]. The rheological parameters \( k \) and \( n \) for any system must, of course, depend on the magnetic field intensity. The magnetorheological effect in a dispersion system used as the working medium for a "dry-fluid" coupling consists of reversible changes in these parameters incurred by the application of a magnetic field. In a constant magnetic field both rheological parameters depend on the physical properties of the dispersion, especially of its solid phase.

According to the shape of its active surfaces, a "dry-fluid" coupling can be of the face, the sleeve, or the hybrid type. In face couplings, the torque is transmitted from the driver disk to the follower disk, both coupled mechanically through the working medium. In sleeve couplings, the working medium fills the gap between two concentric cylindrical surfaces. In hybrid couplings, the surfaces of flat disks and of cylindrical sleeves are active.

We will first determine the torque transmitted across a pair of active surfaces (driver and follower) in a face coupling. Let us assume that one disk (the follower) is stationary while the other (the driver) rotates at a certain angular velocity \( \omega \) (Fig. 1). The gap between both disks is \( \delta \). The hermetically sealed space between the disks contains a magnetorheological medium the behavior of which is described by Eq. (1). In cylindrical coordinates \( r, \varphi, z \) (the Oz axis being perpendicular to the planes of the disks), as long as \( \delta \ll R_1 \), we may assume with sufficient accuracy that \( v_r = v_z = 0 \) and \( v_\varphi = f(r, z) \). The continuity condition with respect to strains will then be satisfied identically.

The differential equations become

\[ \frac{\partial \rho}{\partial r} = \frac{\rho_0}{r} \frac{\partial v_\varphi}{\partial r}, \quad \frac{\partial \rho_{\varphi z}}{\partial z} + \frac{2\rho_{\varphi z}}{r} = 0. \]  \hspace{1cm} (2)

The intensity of strain rates is

\[ h = \sqrt{\left( \frac{\partial v_\varphi}{\partial r} - \frac{v_\varphi}{r} \right)^2 + \left( \frac{\partial v_\varphi}{\partial z} \right)^2}. \]

The first term under the square root will be assumed equal to zero. Considering that within the range \(0 \leq z \leq \delta\)

\[
\frac{\partial \psi}{\partial z} > 0,
\]

we have

\[
h = \frac{\partial \psi}{\partial z}.
\]

Moreover,

\[
\rho_{\psi} = 0, \quad \rho_{\psi z} = kh^{n-1} \left( \frac{\partial \psi}{\partial z} \right) = k \left( \frac{\partial \psi}{\partial z} \right)^n.
\]

Inserting this into (2) yields

\[
k n \left( \frac{\partial \psi}{\partial z} \right)^{n-1} \frac{\partial^2 \psi}{\partial z^2} = 0.
\]

After integration,

\[
\psi = u(r) z + M(r).
\]

In the physical sense

\[
\psi \bigg|_{z=0} = 0 \text{ and } \psi \bigg|_{z=\delta} = \omega \delta,
\]

whence

\[
M(r) = 0, \quad u(r) = \frac{\omega \delta}{\delta} \text{ and } \psi = \frac{\omega \delta}{\delta} z.
\]

For the shear stress \(\rho_{\psi z}\) we obtain the following expression:

\[
\rho_{\psi z} = k \left( \frac{\omega \delta}{\delta} \right)^n.
\]

It becomes evident now that stress \(\rho_{\psi z}\) is independent of \(z\), which agrees with the physical conditions of the problem.

The torque transmitted through the working medium is

\[
M_{cr} = \int_{R_1}^{R_2} 2\pi r^2 \rho_{\psi z} dr = \frac{2\pi k \omega^n}{\delta^n (n + 3)} \left( R_2^{n+3} - R_1^{n+3} \right).
\]

For this formula we can see that, when \(\omega = 0\), the torque transmitted by a coupling is zero, i.e., with a working medium which has no elastic limit it is impossible to transmit torque without producing slip between the disks.

The rheological equation for a medium which obeys a power law but has an elastic limit \(\tau_0\) is

\[
\rho_{\psi h} = -\delta_{\psi h} \rho + 2 \left( \frac{\tau_0}{h} + kh^{n-1} \right) \varepsilon_{\psi h},
\]

(generalized Balkley–Herschel equation). Moreover,

\[
\rho_{\psi z} = \tau_0 + k \left( \frac{\partial \psi}{\partial z} \right)^n
\]

and the magnitude of the torque is

\[
M_{cr} = \frac{2}{3} \pi \tau_0 \left( R_2^3 - R_1^3 \right) + \frac{2\pi k \omega^n}{\delta^n (n + 3)} \left( R_2^{n+3} - R_1^{n+3} \right).
\]

Slip between the disks will occur in this case if the resisting torque at the follower shaft is greater than \(M_0\):

\[
M_0 = \frac{2}{3} \pi \tau_0 \left( R_2^3 - R_1^3 \right).
\]