PRINCIPLES FOR THE DESIGN OF TEACHING

ABSTRACT. In this introductory article, after some initial discussion of an appropriate approach to mathematics as a curriculum subject, we sketch a theory for designing teaching, based on mathematical activity, situations, tasks, and interventions, exposing and resolving cognitive conflicts, changes of structure and context, feedback, reflection and review. We next review the main psychological principles underlying this theory, then consider some examples of teaching designs in the light of the theory. Thus we open the discussion of the theme of this issue, which continues with the fuller discussion of other examples in the remaining articles.

INTRODUCTION

Aims of Mathematical Education

Mathematics arises from the attempt to organize and explain the phenomena of our environment and experience. It has been expressed thus:

Mathematics is ... an activity of organising fields of experience.

H. Freudenthal (1973, p. 123)

Mathematics concerns the properties of the operations by which the individual orders, organises and controls his environment.

E. A. Peel (1971, p. 157)

These descriptions are somewhat non-specific, though Peel is clearly referring to basic mental operations such as classifying, comparing, combining, representing. A more specific characterization of mathematics is given by Gattegno:

To do mathematics is to adopt a particular attitude of mind in which what we term relationships per se are of interest. One can be considered a mathematician when one can isolate relationships from real and complex situations and later on when relationships can be used to create new situations in order to discover further relationships.

Teaching mathematics means helping one’s pupils to become aware of their relational thought, of the freedom of the mind in its creation of relationships; it means encouraging them to develop a liking for such an attitude and to consider it as a human richness increasing the power of the intellect in its dialogue with the universe. (1963, p. 55)

Other authors have offered more controversial descriptions, which emphasize the logical aspects. One may recall Russell’s “the subject in which we never know what we are talking about, nor whether what we say is true”. For the purposes of guiding curriculum construction, we find the positive descriptions more helpful.

Education is normally seen as a forward-looking, purposeful activity, the aim of which is to develop pupils’ capacities and knowledge so as to equip them
more fully for adult life. This crude view is modified by the recognition that the
aim of life-enhancement is available to some extent immediately, and indeed
the concept of continuing education assumes that adults too can continue to
derive benefit from educational experiences.

In terms of the mathematical curriculum, one is attempting to give pupils
experiences of organizing and interpreting significant areas of their experience
by the use of mathematical ideas and activities in a way which equips them to
continue to do this in adult life. It follows that classroom activity should reflect
the way in which mathematical experiences arise in adult life as well as provid-
ing genuine mathematical experiences for pupils in their own immediate situa-
tion. This does not, of course, rule out important ancillary activities, such as the
memorizing of important data or the practising of frequently needed skills, but it
does set them in place as subsidiary to the main mathematical activity of
inquiry.

In designing lessons and building a curriculum, one needs to consider three
aspects: the nature of the mathematical activity, the conceptual content, and the
nature of learning.

**Mathematical Activity**

Most uses of mathematics involve a cycle of mathematization, manipulation,
and interpretation — that is, recognizing in the given situation the relevance of
some mathematical relationship, expressing this relation symbolically, manipu-
lating the symbolic expression to reveal some new aspect, and interpreting this
new aspect or giving some fresh insight into it in the given situation. A relative-
ly complex case is the use differential equations for measuring the vibrations of
a string leading to the prediction of a set of normal modes of vibration — a fun-
damental and a sequence of harmonics. A more elementary case is the use of the
concept of multiplication and the corresponding algorithm to determine the cost
of a certain weight of goods at a given unit price.

It may happen that the transformation of the mathematical representation,
which is the middle part of this cycle, gives rise to some new relationships or pro-
cedures which can then take their place in the body of mathematical knowledge.
In the examples quoted, this may consist of new methods of solving the differen-
tial equations or a recognition of the inverse nature of multiplication and divi-
sion. We might think of the former type of activity — the making and using of a
mathematical representation of reality — as the typical *applied* mathematical
activity, and the latter as pure mathematics. (The Dutch workers use the terms
"horizontal" and "vertical" mathematization for these processes.)

Traditional mathematics instruction has assumed that the part of this process