TWO-DIMENSIONAL ORDERS OF FINITE REPRESENTATION TYPE

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Maximal orders of finite representation type over complete local rings of dimension two and of characteristic zero are classified. This completes the classification begun in [1] for the case that $R$ is a power series ring in two variables.

Let $k$ be an algebraically closed field of characteristic zero, and let $R$ be a normal, complete local $k$-algebra of dimension two, with residue field $k$. Let $K$ be the field of fractions of $R$. By maximal order over $R$ we mean a maximal order in a central simple $K$-algebra. This paper studies maximal orders $R$ which are of finite representation type, meaning that the isomorphism classes of indecomposable Cohen-Macaulay $R$-modules form a finite set. The maximal orders of finite representation type were classified in a previous paper [1] when $R$ is a power series ring, and the last section of that paper attempts to extend the classification to other rings. As van den Bergh pointed out to us, the last section contains an error. The object here is to correct the error and complete the classification. This is done in
section 2. Section 1 contains two results about the Brauer group of $K$ which may be of independent interest: a proof that the index of a division ring $D$ over $K$ is equal to its period, and a description of the Brauer group of $K$ in the case that the exceptional fibre of the resolution of the singularity of $R$ is a tree of rational curves.

Suppose given a maximal $R$-order $A$ of finite representation type in a central division ring $D$ over $K$. We assume that $D \neq K$. By Corollary (2.5) of [1], $R$ is also of finite representation type, and is therefore a quotient singularity: $R = R_1^{G_0}$, where $R_1 = k[u,v]$ and $G_0$ is a small subgroup of $G_2$ [7]. Denote by $A_1$ the ring $R_1 \tilde{\otimes}_R A$, where $\tilde{\otimes}$ denotes the reflexive hull of the tensor product. This ring is also of finite representation type. For, it satisfies the conditions of [1,(2.7)], and in fact it is the ring referred to in Theorem (2.15) of [1]. So Proposition (2.12) of [1] implies that it is of finite representation type. It has the standard form [1, (2.14)] of a hereditary order at every height one prime of $R_1$. The error in [1] is that $A_1$ is assumed to be a maximal order, and Theorem (6.1) is correct only when this is added as a hypothesis. The remaining cases will be treated here by similar methods.

The classification of orders of finite representation type has also been done by Reiten and van den Bergh [12], using almost split sequences.