Microscopic study of the low-energy $^3\text{He}(^3\text{He}, 2p)^4\text{He}$ and $^3\text{H}(^3\text{H}, 2n)^4\text{He}$ fusion cross sections

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We have calculated the $^3\text{He}(^3\text{He}, 2p)^4\text{He}$ and $^3\text{H}(^3\text{H}, 2n)^4\text{He}$ reaction cross sections at low energies within the microscopic multichannel resonating group method. For both reactions, we find good agreement with experiment. For the $^3\text{H}(^3\text{H}, 2n)^4\text{He}$ reaction, our calculated energy dependence reproduces that of each individual low-energy experimental data set, except for a normalization constant. Using this fact, we derive at a low-energy $^3\text{H}(^3\text{H}, 2n)^4\text{He}$ rate by taking the averaged mean of these fits.

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The $^3\text{He}(^3\text{He}, 2p)^4\text{He}$ reaction is an important part within the network of hydrogen burning in stars being the dominant source for the production of $^4\text{He}$ [1]. This reaction as well as its mirror reaction $^3\text{H}(^3\text{H}, 2n)^4\text{He}$ are believed to have also played a role within the nucleosynthesis in the early epoch of our universe [2, 3]. Additionally, the fusion of two tritons via the $^3\text{H}(^3\text{H}, 2n)^4\text{He}$ reaction is one of the key nuclear reactions studied in muon-catalyzed fusion [4].

Various experimental groups have studied the $^3\text{He}(^3\text{He}, 2p)^4\text{He}$ reaction at low energy [5, 6]. As these measurements basically agree with each other, it is generally believed that its reaction cross section at astrophysically important energies is rather well known. However, to derive the cross section at these astrophysically most effective energies ($E \approx 20$ keV for temperatures at the solar core), one has to extrapolate the measured cross section down in energy [1]. Due to the lack of reliable theoretical guidelines, this extrapolation has been performed simply using a quadratic polynomial [6]. There have been several experimental attempts to measure the low-energy $^3\text{H}(^3\text{H}, 2n)^4\text{He}$ cross section. However, it has been pointed out [7] that some of these measurements might be subject to large systematic errors. The $^3\text{H}(^3\text{H}, 2n)^4\text{He}$ data have been parametrized by Hale et al. within an R-matrix formulation [8] which at low energies is dominated by the Los Alamos data [9] having by far the smallest experimental uncertainties.

To our knowledge there exists no theoretical study which calculates the absolute cross sections for either of the two reactions at low energies. In this paper we want to report about the first microscopic studies of the low-energy $^3\text{He}(^3\text{He}, 2p)^4\text{He}$ and $^3\text{H}(^3\text{H}, 2n)^4\text{He}$ reactions performed within the framework of the coupled-channel resonating group method (RGM).

A rigorous study of the two reactions requires the consideration of a 3-cluster scattering state in the outgoing channel. However, in our approach we will follow Sünkel [10] who viewed the reaction as a two-step process: After formation of the compound nucleus, the system will firstly decay into an $\alpha$ particle and a 2-nucleon cluster. The latter, which is energetically unbound, will then finally decay into two nucleons; this, however, is expected to occur outside the range of the nuclear forces.

We will in the following neglect possible D-state admixtures in the cluster wave functions. Then, the two $^3\text{H}^3\text{H}$ clusters may be coupled to total spins $S=0$ and 1. To fulfill the requirements of the Pauli principle, the relative orbital angular momentum in the $S=0$ state must be even, while it is odd for $S=1$. Similarly, the total spin of the $^4\text{He}+2n$ wave function might be either 0 or 1 due to the possible spin couplings of two nucleons. However, the $S=1$ component requires an odd relative orbital angular momentum between the two nucleons, which corresponds to a rather highly excited configuration. We will therefore neglect this component in our approach. With this restriction, the $^3\text{He}+^3\text{He}^3\text{H}+^3\text{H}$ channels with $S=1$ can only couple to the $^4\text{He}+2p$ ($^4\text{He}+2n$) channels via the spin-orbit force. However, we expect this coupling to be rather unimportant for the low-energy transfer reaction to be studied in this paper as $i)$ it is significantly smaller than the one mediated by the central interaction and $ii)$ it requires at least a $p$ wave in the entrance channel which, at the low energies we are interested in is unfavored compared to $s$ wave scattering by penetrability arguments. To simplify our calculation we have thus also...
neglected the $S=1$ component in the entrance channels. Note that our assumption is supported and justified by the following facts: i) The experimental $^3\text{He}(^4\text{He}, 2p)^4\text{He}$ and $^4\text{He}(^3\text{H}, 2n)^4\text{He}$ data expressed in terms of the astrophysical $S$-factor are approximately constant at low energies. ii) The experimental data of [9] have been derived assuming isotropic angular distributions. Such a behaviour (i) and (ii) is expected for a fusion process dominated by (non-resonant) s-wave penetration in the entrance channel, in agreement with the present assumptions. At first glance our assumption might be in contradiction with the $d(d,p)\,t$ reaction where noticeable $p$-wave contributions are found at energies as low as 100 keV. However, for this reaction one has to consider that, differently to the fusion processes to be investigated here, the nuclear configurations in the entrance and in the outgoing channels can easily be coupled to spin $S=1$ so that the coupling of the $d+d\, p$-wave scattering states to a $p+t$ configuration with the quantum numbers $S=1$ and $L=1$ via the central component in the nucleon-nucleon interaction is possible. Note that, again in contrast to the present fusion systems, the measured astrophysical $S$-factor for the $d(d,p)\,t$ reaction at low-energies decreases with increasing energy. This is a clear signature for the fact that scattering states other than s-waves contribute to the low-energy cross sections. Finally, it might be mentioned that the reaction process for the low-energy $^3\text{H}(^3\text{H}, 2p)^4\text{He}$ reaction in an accelerator experiment is different from that of the muon-catalyzed resonant fusion of the tritium nuclei in the $(t,t\mu)$ molecule where due to molecular quantum numbers the reaction occurs in a relative $p$-wave of the two nuclei [4].

Thus, in our approach we adopt a model space which is spanned by antisymmetrized $^3\text{He}+^3\text{He}($$^3\text{H}+^3\text{H})$ and $^4\text{He}+2\,p ($$^4\text{He}+2\,n$) two-cluster wave functions. Our many-body function reads

$$\psi_i = \sum_{i=1}^{3} \sum_{j=1}^{2} \int d\mathbf{r}_{33} g_{ij}^{(3)}(\mathbf{r}_{33}) \frac{\phi_{ij}^{(3)}(\mathbf{r}_{33})}{r_{33}} + \sum_{i,j} \int d\mathbf{r}_{42} g_{ij}^{(4)}(\mathbf{r}_{42}) \frac{\phi_{ij}^{(4)}(\mathbf{r}_{42})}{r_{42}}$$

(1)

with the basis function

$$\phi_{ij}^{(3)}(\mathbf{r}) = \mathcal{A} \left\{ \left( \phi_3(\mathbf{r}_3) \phi_3^*(\mathbf{r}_3) \right)^{S=0} Y_i(\hat{\mathbf{r}}) \delta\left(r-r'\right) \right\}$$

(2a)

$$\phi_{ij}^{(4)}(\mathbf{r}) = \mathcal{A} \left\{ \left( \phi_4(\mathbf{r}_4) \phi_2^*(\mathbf{r}_2) \right)^{S=0} Y_i(\hat{\mathbf{r}}) \delta\left(r-r'\right) \right\}$$

(2b)

referring to the $^3\text{He}+^3\text{He}($$^3\text{H}+^3\text{H})$ and $^4\text{He}+2\,p ($$^4\text{He}+2\,n$) channels, respectively. Here, $\phi_4$ and $\phi_2$ describe the internal degrees of freedom of the $\alpha$ particle and the $2N$-cluster. Following [10, 11] we have approximated the spatial dependence of these cluster functions by Gaussian formfactors with width parameters $\alpha_4$ and $\alpha_2$, respectively. For the $3N$-nucleon cluster we adopted the two-Gaussian ansatz from [12, 13] (width parameters $\alpha_3^1$ and $\alpha_3^2$). Following these references, the coefficients of the two Gaussians can be determined by diagonalizing the microscopic Hamiltonian in the $3N$-nucleon model space spanned by the two Gaussians. The lower energy eigenvalue and its eigenvector determined this way corresponds to the $3N$-nucleon system. The excited state does not describe a physical state of the $3N$-cluster. The inclusion of such a pseudo-state within the many-body wave function, however, might be viewed as a tractable way of increasing the degrees of freedom within the adopted model space by considering cluster distortion effects. Thus our internal $3N$-cluster function reads

$$\phi_{ij}^{(3)}(\mathbf{r}) = \sum_{j=1}^{2} \beta_j \exp \left\{ -\alpha_j^0 \sum_{k=1}^{3} (\mathbf{r}_k - \mathbf{R}_j)^2 \right\};$$

(3)

$$\mathbf{R}_j = \frac{1}{3} \sum_{k=1}^{3} \mathbf{r}_k$$

with $i=1,2$ referring to the physical ground state of the $3N$ cluster and to the pseudo-state, respectively. Note that due to exchange symmetry the configurations $(i=1,j=2)$ and $(i=2,j=1)$ in (1) are referring to the same channel and will only be considered once in our calculation.

The relative functions $g_{ij}$ in the many-body wave function (1) are determined by solving the Schrödinger equation in the model space spanned by the basis function (2):

$$\langle \phi_{ij}^{(3)}, i(r') | H-E | \psi_i \rangle = 0$$

(4a)

$$\langle \phi_{ij}^{(4)}, i(r') | H-E | \psi_i \rangle = 0$$

(4b)

which have to be solved for all $i$, $j$ and $r'$. By using (1) for the many-body function $\psi_i$, (4) can be transformed into a set of coupled integrodifferential equations (RGM equations [11]) which can be solved numerically.

Our microscopic Hamiltonian reads

$$H = \sum_{i=1}^{6} \frac{\mathbf{p}_i^2}{2m} + \sum_{i<j} v_{ij} - T_{cm}.$$  

(5)

Consistent with the discussion given above, we have neglected contributions to the $N\,N$ interaction $v_{ij}$ other than the central component and the Coulomb force. For the central $N\,N$ interaction we adopted the Minnesota potential [14] which has been extensively used in studies of light nuclear systems. For our choice of interaction the parameters within the cluster function read $\alpha_2 = 0.36 (0.36)\,\text{fm}^{-2}$, $\alpha_3 = 0.514\,\text{fm}^{-2}$, $\alpha_3^1 = 0.183 (0.191)\,\text{fm}^{-2}$, and $\alpha_3^2 = 0.63 (0.639)\,\text{fm}^{-2}$, while we find the coefficients $\beta_1^0 = 0.114 (0.120)$ and $\beta_1^0 = 0.859 (0.879)$ for the ground states of the $3N$ cluster and $\beta_1^0 = -0.247 (-0.267)$ and $\beta_1^0 = 1.51 (1.57)$ for the $3N$ pseudostates. Note that the numbers given in parentheses refer to the $^3\text{H}+^3\text{H}$ and $^4\text{He}+2\,n$ systems. With this choice of parameters the binding energies and rms-radii of the various clusters are reasonably well reproduced [11-13]. The expectation value of the fictitious $2p$-cluster is calculated as $\sim 5.5$ MeV, while it is $\sim 4.8$ MeV.