A Semiclassical Approach to Pairing Vibrations and Pairing Correlation Effects on Giant Resonances

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We extend the semiclassical approaches to the dynamics of nuclear collective motions, based on the Wigner transform of quantum mean field theories, to the inclusion of pairing correlation effects. We develop simple analytic equations for the contributions to giant resonance frequencies, which are in general quite small.

We are able also to study pairing vibrations, related to oscillations of the superfluid part. Hydrodynamics-like solutions are obtained, without distortion of the Fermi surface, corresponding to low energy excitations.

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1. Introduction

The foundations of the equations of motion for a self-consistent nuclear fluid dynamics can be derived in a clear way from the Wigner transform of the Time Dependent Hartree-Fock (TDHF) equations [1-5]. In recent years a linearized version of the nuclear Vlasov equation obtained in this way has been widely used to study nuclear collective motions. In particular it has been shown that solutions derived just scaling in phase space the nuclear distribution function are reproducing quite well isoscalar and isovector giant resonances [6, 7]. In the present work we generalize the approach to include explicit pairing correlation effects. This extension is not trivial since the two particle distribution function is in general complex and we can establish a scaling behaviour only for its absolute value, as it will be clear in the article.

We show that pairing effects on the structure of giant resonances are quite small, as expected since major shell excitations are involved. However our semiclassical analysis allows also the study of oscillations for the abnormal density, interpreted as pairing vibration modes. We find low frequency eigenmodes which are consistent with usual hydrodynamical solutions of the superfluid equations of motion without distortion of the Fermi surface [8, 9].

The starting point is a self-consistent local approximation to the Time Dependent Hartree-Fock-Bogoliubov (TDHFB) equations which is similar to the nuclear fluid dynamics approach [10, 11] taking into account the superfluid internucleon interaction.

In Sect. 2 we discuss some general formalism and the structure of the equilibrium solutions. In Sect. 3 we solve the problem for oscillations of the normal part of the nuclear fluid starting from a variational approach. Section 4 is devoted to the collective motions of the superfluid part. Finally some brief conclusions are drawn in Sect. 5.

2. Nuclear Fluid Dynamics with Pairing Correlations: The Equilibrium Solution

It is well known that the Vlasov equation represents the semiclassical limit, through the Wigner transform of the one-body density matrix, of a mean field dynamical theory [1-5]. A set of fluid-dynamical equations of motion for the local density, current, pressure tensor and so can be obtained from the Vlasov eq. for the distribution function just performing suitable p-moment projections. A truncation can be automatically achieved by assuming a low multipolarity distor-
tion for the momentum distribution. Indeed this is the method used to study Giant Resonances as solutions of the linearized Vlasov equation in the scaling approximation [6, 7]. Unfortunately such procedure cannot be used in a direct way in the case of TDHFB equations of motion because the correlation distribution function contains an imaginary part. However we will show that it is possible to overcome this difficulty just directly using the variational principle for the action integral.

The total energy of the system in the Hartree-Fock-Bogoliubov (HFB) approximation can be written in the form [11]:

$$E = \text{Tr} \left[ \left( \frac{\hat{p}^2}{2m} + \frac{1}{2} \hat{r}^2 \right) \rho + \frac{1}{2} \hat{\Delta} \hat{K}^+ \right] ,$$  \hspace{1cm} (2.1)

where

$$\hat{\rho} \equiv \rho_{\sigma\sigma'}(r, r', t) \equiv \langle \tilde{\psi}_{\sigma'}(t', t) \tilde{\psi}_{\sigma}(t, t) \rangle$$  \hspace{1cm} (2.2)

is the one-body density matrix. If the spin-orbit interaction is neglected we have

$$\rho_{\sigma\sigma'}(r, r', t) = \delta_{\sigma\sigma'} \rho(t, r', t)$$  \hspace{1cm} (2.3)

and consequently a self-consistent mean field

$$\hat{f} \equiv \delta_{\sigma\sigma'} \hat{V}(r, r', t)$$  \hspace{1cm} (2.4)

expressed through the density matrix and the nucleon-nucleon interaction in particle-hole channel $v^{(ph)}$ as

$$V(r, r', t) = \int \text{d}^3 r' v^{(ph)}(r, r', t) \rho(t, r', t) \delta(r-r')$$  \hspace{1cm} (2.5)

(direct term: the extension to the exchange part is straightforward).

The pairing correlation density matrix $\hat{k}$ is given by

$$\hat{k} \equiv k_{\sigma\sigma'}(r, r', t) \equiv \langle \tilde{\psi}_{\sigma'}(t', t) \tilde{\psi}_{\sigma}(t, t) \rangle.$$  \hspace{1cm} (2.6)

In absence of spin-orbit interaction we get

$$k_{\sigma\sigma'}(r, r', t) = g_{\sigma\sigma'} k(t, r', t), \parallel g_{\sigma\sigma'} \parallel = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$  \hspace{1cm} (2.7)

The pairing mean field

$$\hat{\Delta} \equiv \Delta_{\sigma\sigma'}(r, r', t) = g_{\sigma\sigma'} \Delta(t, r', t)$$  \hspace{1cm} (2.8)

is determined by the nucleon-nucleon interaction in the particle-particle channel $v^{(pp)}$ as

$$\Delta(t, r', t) = \frac{1}{2} v^{(pp)}(r, r') k(t, r', t).$$  \hspace{1cm} (2.9)

In a general case the time dependent correlation density matrix $k(t, r', t)$ and the pairing mean field $\Delta(t, r', t)$ are not real values.

In a formal sense the expression (2.1) is not connected to the assumption of the existence of a superfluid condensate and it can be considered as an extension of the Hartree-Fock approximation to a more general non-determinant probe function.

Summing up (2.1) over the spin index, after the Wigner transformation, in the $\hbar \to 0$ limit, the following semiclassical expression for the total energy can be written:

$$E = \int \text{d}^3 r \int \text{d}^3 p \left[ 2 \left( \frac{p^2}{2m} + V_w \right) \rho_w - \frac{1}{2} (\Delta_w k^*_w + \Delta^*_w k_w) \right].$$  \hspace{1cm} (2.10)


From the time-reversal invariance, in the static limit both functions $k_w$ and $\Delta_w$ assume real values. Using the variational procedure for the ground state energy (2.10), in this static case the following equations for the equilibrium distributions $\tilde{k}_w$ and $\tilde{\rho}_w$ can be found [12]

$$2 \tilde{k}_w(r, p) \cdot \tilde{\rho}_w(r, p) - 2 \tilde{\Delta}_w(r, p) \tilde{\rho}_w(r, p) + \tilde{\Delta}_w(r, p) = 0$$  \hspace{1cm} (2.11)

$$\tilde{k}_w^*(r, p) = \tilde{\rho}_w^*(r, p) - \tilde{\rho}_w^*(r, p).$$  \hspace{1cm} (2.12)

Equation (2.11) can also be obtained from the HFB equation for the correlation density matrix after the Wigner transformation and in the $\hbar \to 0$ limit. Equation (2.12) comes from the normalization of the generalized density matrix $\tilde{R}$

$$\tilde{R}^2 = \tilde{R}, \hspace{1cm} \tilde{R} \equiv \begin{pmatrix} \tilde{\rho} & \tilde{k} \\ -\tilde{k}^* & 1-\tilde{\rho}^* \end{pmatrix}.$$  \hspace{1cm} (2.13)

Equations (2.11) and (2.12) have solutions which are corresponding to a spherical Fermi surface with non zero diffuseness determined by the energetic gap $\tilde{\Delta}_w$. In particular, for the equilibrium distribution function $\tilde{\rho}_w$ we have [12]

$$\tilde{\rho}_w(r, p) = \frac{1}{2} \left\{ 1 - \frac{p^2}{2m} + V_w - \lambda \right\} \left\{ \left( \frac{p^2}{2m} - V_w - \lambda \right)^2 + \tilde{\Delta}_w^2 \right\},$$  \hspace{1cm} (2.14)

where $\lambda$ is the chemical potential.

3. Small Oscillations in the Normal Part of the Nuclear Fluid

The dynamics of a Fermi system is connected to the time dependent distortion of the Fermi surface. In a semiclassical limit the dynamical distribution func-