QCD sum rules with finite masses

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Abstract. The concept of QCD sum rules is extended to bound states composed of particles with finite mass such as scalar quarks or strange quarks. It turns out that mass corrections become important in this context. The number of relevant corrections is analyzed in a systematic discussion of the IR- and UV-divergencies, leading in general to a finite number of corrections. The results are demonstrated for a system of two massless quarks and two heavy scalar quarks.

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1. Introduction

In the last fifteen years the concept of QCD sum rules invented by Shifman et al. [1] has been very well developed. The intention was to include non-perturbative effects in QCD calculations, which naturally become important when dealing with hadron physics. The concept of vacuum condensates is best illustrated in the case of chiral symmetry, which is spontaneously broken for hadrons as one can see by comparing the masses of different mesons with identical quantum numbers (i.e. \( \rho \) and \( \omega \), mesons). A direct consequence of spontaneous breakdown of any symmetry is the appearance of non-vanishing vacuum condensates. The idea of QCD sum rules is to parametrize the non-trivial QCD vacuum with vacuum condensates, including the chiral condensate \( \langle \bar{q}q \rangle \), and in this way to realize a parametrization of non-perturbative effects in QCD.

The condensates appear as matrix elements in the operator-product-expansion [2] (OPE) of a physical function (i.e. the polarization function):

\[
\Pi(q^2) \sim \sum_{j=1}^{\infty} \langle 0 | B_j(0) | 0 \rangle \int d^4 x E_j(x) e^{i q \cdot x}
\]

This expansion separates perturbative (Wilson coefficients \( E_j(x) \)) and non-perturbative (condensates \( \langle 0 | B_j(0) | 0 \rangle \)) parts, being thus completely calculable with perturbative Feynman rules. It can be related to the hadronic spectrum through a dispersion relation with one subtraction \( C \)

\[
\Pi_{\text{OPE}}(q^2) = \frac{q^2}{\pi} \int_\Lambda \frac{\text{Im} \Pi_{\text{had}}(s)}{s(s-q^2)} + C
\]

where the imaginary part of the function \( \Pi_{\text{had}} \) is connected to the hadronic spectrum. In that way it is possible to get hadronic quantities like the mass of a bound state as a function of a few vacuum condensates, which are fitted to a large number of hadronic data.

The kind of condensates being relevant in the OPE depends strongly on the flavour composition and the quantum numbers of the bound state considered. In the case of light quarks (up or down) there is strong evidence for the presence of quark-antiquark pairs in the vacuum, so that apart from the gluon-condensates the quark-condensates have to be taken into account. If the particles are heavy quarks (charm or heavier) the value of the corresponding condensates becomes very small and may be neglected, so that the calculation may lead to a reasonable result by including the gluon-condensate only. The strange quark, however, poses a problem because it is neither light nor heavy. Therefore its condensate cannot be neglected a priori, and in order to take it into account, it is necessary to treat massive quarks in QCD sum rules – a problem unsolved up to now. We derive several results necessary to approach this goal and elucidate the problems occurring.

This article is organized as follows. We shall illustrate the problem of finite mass sum rules with a specific example (introduced in Sect. 2), a detailed study of which will be published elsewhere. In the third section we give an expression for the matrix elements of normal-ordered, nonlocal field products – which occurs in the expansion of the correlator under consideration –, demonstrating that the finite masses cause corrections to the commonly used expressions. Afterwards some of the Wilson coefficients introduced above will be discussed with special attention to the appearing infrared and ultraviolet divergencies and to the relevance for the sum rule.
2. The system under consideration

For studying the influence of mass terms in QCD sum rules calculations we will consider a four-particle bound state composed of two light fermionic quarks and two heavy scalar quarks. The last ones are hypothetical particles introduced in QCD phenomenologically. Scalar quarks are predicted by the GUT-theory super-symmetry [3] (they are called squarks), such that we are not merely dealing with a toy model. The physical problem motivating this model is whether or not the 2 quark – 2 squark bound state may be energetically more favourable than a 2 squark state, which is predicted to have a very high mass [4]. The lowest lying 2 quark – 2 squark bound state can be calculated – using QCD sum rules – in dependence of different new condensates and of the squark mass. These parameters are unknown, but it one could find a reasonable choice of them, such that the 2 quark – 2 squark bound state mass becomes small, this would be most interesting for supersymmetry phenomenology. The results of these calculations will be published elsewhere.

In the following calculation the quark masses are neglected and the scalar quarks are treated as massive particles. In order to calculate the mass of the lowest lying bound state of this system, the OPE of the polarization-function corresponding to the diagram in Fig. 1 has to be considered [see (1)], which is given by the two-point-function

\[ \Pi(k^2) = -\frac{i g^2}{3} \int \frac{d^4(x-y)}{x-y} \langle 0| T \{ J^\mu(x), J^\nu(y) \}|0 \rangle \]

where \( T \) denotes the time ordered product. The current \( J^\mu(x) \) contains an incoming and outgoing fermionic and scalar quark:

\[ J^\mu(x) = g \bar{\psi}(x)\gamma^\mu \psi(x) \phi(x). \]

The present sum rule is calculated in lowest order of QCD perturbation theory. Thus the exchange of gluons and the coupling to the gluon condensate are neglected.

Usually QCD sum rule calculations are carried out in configuration space. In the case of massless particles this choice simplifies the calculations: the propagators are as simple as in momentum space, but the number of integrations is reduced to one, while in momentum space the number of integrations is equal to the number of loops in the corresponding Feynman diagram. However, in the case of particles with non-vanishing masses the propagators become very cumbersome, so that in most cases calculations in momentum space will be more convenient.

The next step is to find the explicit form of the OPE for the polarization function (3). The time-ordered product has to be expanded into normal-ordered products with attention to the non-vanishing vacuum condensates. In these products the field operators are taken at different points in Minkowski space, while the vacuum condensates of scalar or fermionic fields are defined at the same points. Nevertheless, the nonlocal normal ordered products commonly are identified with the vacuum condensates. We claim that this procedure is correct only in zeroth order in the mass of the particles under consideration.

\[ \langle 0| \text{Tr} \{ S(x-y)\gamma^\mu S(y-x)\gamma^\nu \} \text{Tr} \{ A(y-x)A(x-y) \}|0 \rangle. \]

When expanding expressions like that given in (3) one quite generally encounters nonlocal normal ordered product of scalar or fermionic field operators. In these products the field operators are taken at different points in Minkowski space, while the vacuum condensates of scalar or fermionic fields are defined at the same points. Nevertheless, the nonlocal normal ordered products commonly are identified with the vacuum condensates. We claim that this procedure is correct only in zeroth order in the mass of the particles under consideration.