Estimates of hypernuclear production via the exclusive \((p, K^+)\) reaction

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Abstract. We investigate the possibility of producing hypernuclei with proton beams via \((p, K^+)\) reaction. We present differential cross sections calculations utilizing the distorted wave impulse approximation in momentum space. We consider the reactions \(12\text{C}(p, K^+)\)\(^3\text{C}\) and \(16\text{O}(p, K^+)\)\(^{17}\text{O}\) within the energy region 0.8 GeV–1.2 GeV. We study both the case of formation of \(A\) in S-state \((^{12}\text{C})\) and P-state \((^{16}\text{O})\). We take into account the contribution of both one-step and two-step processes when \(A-K\) pair is produced directly by incoming proton and intermediate pion, respectively. It is found that practically in all cases the two-step processes give significant contribution.

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1. Introduction

The reactions used so far for production of \(A\) hypernuclei are the strangeness exchange \((K^-, \pi^-)\) reaction \([1]\) and pion-induced \((\pi^+, K^+)\) reaction \([2]\). The first type of reaction is due to small momentum transfer at \(0^\circ\) that tends to populate low-spin states, while the reaction \((\pi^+, K^+)\) is necessarily accompanied with momentum transfer larger than Fermi momenta in nuclei and basically leads to the population of high-spin hypernuclear states.

Further progress in hypernuclear physics calls for higher energy resolution than available at present and the use of interaction particles being under some certain conditions (for example, polarization of incoming particles or a target). Higher energy resolution is required for the further study of hypernuclear structure and polarization of interacting particles may enable us to study in much more detail the mechanism of hypernuclear formation, the properties of \(A\)-nuclear mean-field and effective \(A-N\) interactions.

The \(pA \rightarrow BK^+\) reaction seems to have an advantage in these two points \([3]\). However, caused by the very large momentum transfer (\(~1\ \text{GeV/c}\)) the probability of hypernuclear formation becomes very small. Nevertheless high proton beam intensities available at present might help in getting measurable signals of the hypernuclei and therefore proton-induced exclusive kaon production merits further study.

In this paper the exclusive \(pA \rightarrow BK^+\) reaction is studied and the theoretical estimations of the possible cross section are presented.

Our paper is organized as follows. In the next section we outline our formalism and then we discuss the results of our calculations. We conclude with suggestion for possible future calculations of similar processes.

2. Formalism

As it was mentioned above, the exclusive \(A(p, K^+)B\) reaction is characterized by very large momentum transfer \((~1\ \text{GeV/c})\). In the naive plane wave approximation the cross section is proportional to the square of \(A\)-particle wave function in discrete state in momentum space. In its turn, it leads to the significant suppression of the cross section. However it is natural to suggest that the inclusion of intermediate processes which by itself have much larger cross sections, might give rise to the increase of the cross section of the reaction \(A(p, K^+)B\). Moreover, the inclusion of such “few-step” processes implies some “sharing” of the total momentum transfer and it also can be effectively lead to the additional growth of the cross section. It is worth mentioning that such “few-step” contributions were found to be important, for example, in the exclusive processes \(^3\text{He}(\pi, \eta)\)\(t\) \([4]\), \(A(p, \pi)A + 1\) \([5]\), and in inclusive reaction \(A(p, \eta)X\) \([6]\).

In this paper we consider both the one-step contribution when the \(K^+A\) pair is ejected directly from the incoming nucleon and the two-step contribution when the \(K^+A\) pair production is produced in the result of the process.
\[ p \to \pi \to K. \] The reaction of pion production (the corresponding cross section is about 10 nb/ster [7]) in forward direction for light nuclei at the kinetic energy of the initial proton \( \sim 1 \text{ GeV} \) whereas the second step is the pion-induced hypernuclei production with \( \text{d} \sigma / \text{d} \Omega \sim 10 \text{ mb/ster} \) in forward direction [8]. The typical momentum transfers are 600–650 MeV/c and 300–350 MeV/c for the reactions \( A(\pi, \pi')A + 1 \) and \( A(\pi, K^{-})B \) correspondingly. Since the cross section of the reaction \( A(\pi, \pi')A + 1 \) drops down very fast with the growth of the angle the main contribution is that from pions, going in the forward direction. The cross section for the \( A(p, K^{+})B \) reaction is written as
\[
\frac{\text{d} \sigma}{\text{d} \Omega} = \frac{1}{(2\pi)^2} \left[ \frac{p}{E_{\text{tot}}} \right] \frac{1}{1} \sum \left| T_{pK} \right|^2
\] (1)

In Eq. (1), \( p, p' \) are the momenta of the initial nucleon and the final kaon; \( E_p, E_{\text{tot}}, E_f \) are the energies of the initial nucleon, initial nucleus, final nucleus and total energy, respectively. In Eq. (1) \( \sum_{2s}^{2s} \sum_{2s+1}^{2s+1} \) represents an average over initial spins and sum over final spin state in this reaction.

The quantity \( T_{pK} \) in Eq. (1) corresponds to the transition matrix element for the process
\[ p(p) + A \to K(pK) + A B. \]

We evaluate this DWIA in momentum space where the amplitude can be written as follows
\[
T_{pK}(\vec{p}, \vec{p}_k) = V_{pK}(\vec{p}, \vec{p}_k) + \sum_f \left( \frac{d\vec{q}}{(2\pi)^3} \right) T_{\pi K}(\vec{q}, \vec{p}_k) G_0(E_{\pi'}, q) V_{\pi\pi}(\vec{q}, \vec{q}_k, E_{\pi})
\] (2)

In Eq. (2), the first term corresponds to the contribution of the direct emission of the \( \Lambda K \) pair from the incoming proton. The second term represents the two-step process when intermediate pion-induced hypernucleus production is followed by the formation of pion itself as the result of the proton-nucleus interaction. In second term we include both production of \( \pi^{\pm} \) and \( \pi^0 \) pions from the initial nucleus.

The first term in Eq. (2) has the form:
\[
V_{pK}(\vec{p}, \vec{p}_k) = \langle \vec{p}_k, A | H_{\Lambda KN} | \psi_{\pi^+}(A) \rangle
\] (3)

Here \( \psi_{\pi^+} \) is the incoming proton distorted wave function and \( H_{\Lambda KN} \) is the vertex of the \( N \to \Lambda K \) transition which in the simplest non-relativistic version can be written as follows
\[
H_{\Lambda KN} = g_{\Lambda KN} \vec{\sigma} \left( \frac{\vec{p}_k}{2M_{\Lambda}} \right)
\] (4)

Here \( M_{\Lambda} \) is the mass of \( \Lambda \); \( g_{\Lambda KN} \) is the \( \Lambda KN \) coupling constant. On the choice of the value of coupling constant \( g_{\Lambda KN} \) there is no definite agreement. In our calculation we used the value taken from Ref. [9] \( g_{\Lambda KN} = 14 \).

The Eq. (3) can be written in the following form
\[
V_{pK}(\vec{p}, \vec{p}_k) = g_{\Lambda KN} \int d\vec{q} \psi_{\pi^+}(\vec{q}) e^{-i\vec{q} \cdot \vec{p}_k} \psi_{\pi^+}(\vec{q})
\]
\[
\cdot \left( s, m_{\pi^+} \right) \left( \frac{\vec{p}_k}{2M_{\Lambda}} \right) \left( s, m_{\pi^+} \right)
\] (5)

In Eq. (5) \( |s, m_{\pi^+} \rangle \), and \( |s, m_{p_\pi} \rangle \) are the spin functions of incoming nucleon and \( \psi_{\pi^+}(\vec{q}) \) and \( \psi_{\pi^+}(\vec{r}) \) are the wave functions of \( \Lambda \) in some discrete state with quantum number \( e_{\pi}, m_{\pi} \) and of incoming nucleon (distorted nucleon wave). Due to smallness of the spin-orbital part of the \( \Lambda \)-nucleus potential we neglect its contribution in Eq. (5).

The wave function of \( \Lambda \) is calculated using the \( \Lambda \)-nucleus potential [10] \( U_{\Lambda K}(r) = -U_0 f(r) \) where \( U_0 = 28 \text{ MeV} \) and function \( f(r) \) has the Woods-Saxon form with the parameters \( r_0 = 1.128 + 0.439 A^{-1/3} \text{ fm}, a = 0.54 \text{ fm} \). The function \( \psi_{\pi^+}(\vec{r}) \) describing the distortion of the incoming proton in nuclear field is obtained by numerical solution of the Schrödinger equation using the optical potential from Ref. [11].

One notes that we neglect the effect of the kaon distortion. Taking into account the relative weakness of kaon-nucleus interaction this assumption seems to be quite reasonable for calculations which are supposed to give just the correct order of the magnitude of the cross section.

The Eq. (5) was calculated numerically. The pion Green function can be decomposed using the standard expression
\[
\frac{1}{p_{\pi}^2 - m_{\pi}^2 \pm ie} = P \frac{1}{p_{\pi}^2 - m_{\pi}^2} \pm iz \delta(p_{\pi}^2 - m_{\pi}^2)
\] (6)

where on-shell pions corresponds to the \( \delta \)-functional part of Eq. (6) and the principal value integral describes off-shell pions. As it can be seen from Eq. (7) the pion-nucleus rescattering is included in the operator \( T_{\Lambda K} \). To calculate the effects of the off-shell pion distortion we assumed the following form of the corresponding amplitude \( T_{\pi \pi} = T^{\text{os}}(k, k', E) \), where \( T^{\text{os}} \) is the fully on-shell pion-nucleus rescattering amplitude, and the factor \( F(k, k', E) \) defines its off-shell extension. We assume the function \( F(k, k', E) \) has a separable form \( F(k, k', E) = \nu(k)\nu(k')/\nu(k)^2 \) where \( \nu(k) = (1 + \lambda k^2)^{-1}, \lambda = 0.224 \text{ fm}^2 \). In our calculations off-shell effects in pion rescattering were found to be relatively small.

In the DWIA in momentum space \( T_{\pi K} \) is given as follows
\[
T_{\pi K}(\vec{p}_\pi, \vec{p}_k) = V_{\pi K}(\vec{p}_\pi, \vec{p}_k)
\]
\[
+ \int \frac{d\vec{q}}{(2\pi)^3} V_{\pi K}(\vec{q}, \vec{p}_k) G_0(E_{\pi'}, \vec{q}) T_{\pi\pi}(\vec{q}, \vec{p}_\pi)
\] (7)

In Eq. (8) \( t(11N \to AK) \) is the \( t \)-matrix for the elementary reaction \( \pi N \to AK \). To evaluate Eq. (8) we used a factorization approximation in which the elementary amplitude is evaluated at some effective Fermi momentum
\[
\langle \vec{p}_\pi | t(11N \to AK) \rangle, \langle A, p_\pi \rangle
\]
\[
= F^{1/2} t(w_{\pi}, p_\pi, \vec{p}_\pi, \vec{p}_\pi) \cdot \int d^3r \bar{\psi}(\vec{r}) \psi_{\pi^+}(\vec{r}) \exp \left[i \vec{Q} \cdot \vec{r} \right]
\] (9)

In Eq. (9) the factor \( F^{1/2} = \exp [\vec{Q}^2/4A\lambda] \) is the standard correction which compensates for the lack of translation.