Analysis of proton elastic scattering from $^{16}\text{O}$ and $^{40}\text{Ca}$ at 800 MeV within the $\alpha$-particle model

Ruan Wenying $^1$, Liu Youyan $^2$

$^1$ Department of Applied Physics, South China University of Technology, Guangzhou 510641, PR China
$^2$ CCAST (World Laboratory), P.O. Box 8735, Beijing 100080, PR China and Department of Applied Physics, South China University of Technology Guangzhou 510641, PR China

Received: 15 December 1993 / Revised version: 21 March 1994

Abstract. Elastic scattering of proton from $^{16}\text{O}$ and $^{40}\text{Ca}$ are analyzed within the $\alpha$-particle model. The results are in good agreement with the experimental data. The role displayed by the phase factor of $p-\alpha$ scattering amplitude is investigated.

PACS: 24.40; 24.10.Ht

1. Introduction

In the past ten years or so, considerable theoretical efforts have been devoted to the investigation of polarization phenomena in proton-nucleus scattering at GeV energies. Some studies used the relativistic impulse approximation (RIA) model, others used the nonrelativistic KMT model [1]. In the later case the second order effects were generally included either phenomenologically or through approximations. It has been found that various effects, such as the Fermi motion, Pauli correlations, short range dynamic correlations and target nucleon spin-flip etc. are important and should not be neglected. However, in the nucleon model where using the $p-N$ amplitude as the basic input, these effects are very difficult to comprehensively deal with.

For some light even-even nuclei which can be regarded as being made up of $\alpha$-clusters, one can regard the $\alpha$-particles in the nuclei as the scatterers and utilize the $p-\alpha$ amplitude as the basic input in the theoretical calculations. In this formalism, the various effects mentioned above would be “automatically” included to some extent in the $p-\alpha$ amplitude obtained from fitting the experimental data. Specifically, since the $\alpha$-particle is spin-0, the spin effects can be completely included. Therefore calculations based on the $\alpha$-particle model are expected to produce good fit to the experimental data.

In a series of previous papers [2-4], we have used the independent $\alpha$-particle model to analyze the elastic scattering of proton from light even-even nuclei at 200 MeV and around. The framework adopted was either the KMT optical potential or the Glauber diffractive model. The elementary $p-\alpha$ scattering amplitude used in the calculations were obtained from the experimental data by phenomenological parametrization. Due to the lack of spin rotation parameter $Q$'s data for $p-^4\text{He}$ scattering, uncertainties in the elementary amplitude were then inevitable. This prevented us from arriving at definite conclusions. Recently, experimental $Q$ data for $p-^4\text{He}$ scattering at 800 MeV has been available [5]. This together with the analyzing power and the differential cross sections already measured [6], enables us to determine the $p-\alpha$ amplitude to within an overall phase (we will return to this phase later) empirically. The differential cross section $d\sigma/d\Omega$, analyzing power $A_y$ and spin rotation parameter $Q$ for proton elastic scattering from $^{16}\text{O}$ and $^{40}\text{Ca}$ have also been measured precisely at 800 MeV [7, 8]. Consequently, this constitutes strict tests to the independent $\alpha$-particle model. Moreover, the 800 MeV incident energy is high enough to make the Glauber-type calculation more reliable.

In this paper, the Glauber multiple scattering theory is used to generate the proton-nucleus scattering amplitude. The target nuclei are described by the independent $\alpha$-particle model. As is well known, the most general theory for clusterization in nuclei is the resonating group model. However, the wavefunctions obtained from this model is complicated and not ready for applications in the calculation of high energy collisions. Therefore, the wavefunctions based on the rigid $\alpha$-particle model have been more extensively used [9]. The drawback of these wavefunctions is that the multi-integrations in the high order terms can not be carried out analytically and further approximations are needed. With the independent $\alpha$-particle model, the integrations can be carried out analytically to all orders.

2. Proton-nucleus scattering amplitude

Previously, we have proposed a parametrization of the following expression [2]
\[ f_{p\alpha}(q) = f_1(0)(1 - q^2/t_1)(1 - q^2/t_2)e^{R_2q^2/2} + f_2(0)(1 - q^2/t_3)(1 - q^2/t_4)e^{R_3q^4/2} q(\mathbf{n} \cdot \mathbf{\sigma}) \]  (1)

to fit very well the \( p - \alpha \) scattering data at 200 MeV and around [10].

In the above expression, \( t_i \) are complex parameters, \( R_j \) are real parameters, \( f_i(0) \) are the forward amplitudes of the corresponding terms. This is somewhat similar to the parametrization of Binon using for pion-\( \alpha \) scatterings at hundreds MeV [11]. However, we found from our fitting work that such a parametrization was unable to reproduce the “saw-tooth” shape of the spin rotation parameter observed at 800 MeV [5].

In a recent paper, Berezhnoy et al. have proposed another parameterization for the \( p - \alpha \) amplitude [12], which is composed of two Gaussians for the spin-independent and spin-dependent parts, respectively given by

\[ f_{p\alpha}(q) = f_1(q) + f_2(q) \sigma \cdot \mathbf{n} \]  (2)

with

\[ f_1(q) = k[G_1e^{-\beta_1q^2} + G_2e^{-\beta_2q^2}] \]  (3)

\[ f_2(q) = k[G_3e^{-\beta_3q^2} + G_4e^{-\beta_4q^2}] q \cdot \]  (4)

They required the parameters in the above expressions to satisfy the following relations

\[ G_2 = \frac{3iG_1^2}{32\beta_1}, \quad G_4 = \frac{3iG_1G_3\beta_3}{8(\beta_1 + \beta_2)^2} \]

\[ \beta_2 = \frac{\beta_1}{2}, \quad \beta_4 = \frac{\beta_1 \beta_3}{\beta_1 + \beta_3} \]

such that the second terms in each part corresponding to the leading contribution coming from the double \( p - N \) scattering amplitude when the \( p - \alpha \) amplitude is deduced from \( NN \) scattering amplitude with single Gaussian density for the ground state of the \( ^4He \) nucleus. The values of these parameters are listed in Table I, which fit the experimental data taken at 800 MeV to the maximum of momentum transfer \( q \leq 3 \) fm.

Following the Glauber theory, the \( p \)-nucleus scattering amplitude can be expressed as [4]

\[ F_{pN}(q) = \frac{iK_{pN}}{2\pi} \int \exp(iq \cdot \mathbf{b})d^2\mathbf{b} \]

\[ \times \{1 - \exp[i\chi_c(b) + i\chi_N(b)]\} \]  (5)

where \( \chi_c \) and \( \chi_N \) are respectively the Coulomb and the nuclear phase shift function. With the help of

\[ 1 - \exp[i\chi_c(b) + i\chi_N(b)] = 1 - \exp[i\chi_c(b)] \]

\[ + \exp[i\chi_c(b)][1 - \exp(i\chi_N(b))], \]  (6)

(1) can be divided into two parts

\[ F_{pN}(q) = F_c(q) + F_N(q) \]  (7)

where

\[ F_c(q) = \frac{iK_{pN}}{2\pi} \int \exp(iq \cdot \mathbf{b})d^2\mathbf{b} \]

\[ \times \{1 - \exp[i\chi_c(b)]\} \]  (8)

\[ F_N(q) = \frac{iK_{pN}}{2\pi} \int \exp(iq \cdot \mathbf{b})d^2\mathbf{b}\exp[i\chi_c(b)] \]

\[ \times \{1 - \exp[i\chi_N(b)]\} \]  (9)

are respectively the Coulomb amplitude and the nuclear amplitude when there exists the Coulomb interaction.

2.1. The Coulomb phase shift function

In treating the Coulomb interaction, the incident proton is regarded as a point-like charge particle while the target nucleus as a uniform charge sphere with radius \( R_c \). This leads to

\[ V_c(r) = \begin{cases} -\frac{3}{2} \left(1 - \frac{r^2}{3R_c^2}\right) \frac{Ze^2}{R_c}, & r < R_c, \\ \frac{Ze^2}{r}, & r \geq R_c. \end{cases} \]  (10)

and the Coulomb phase shift function

\[ \chi_c(b) = \begin{cases} 2v \{\ln(k_{pN}R_c) & b \leq R_c, \\ + \ln[1 + (1 - b^2/R_c^2)^{1/2}] & b > R_c \end{cases} \]  (11)

\[ -\frac{1}{2}(1 - b^2/R_c^2)^{3/2}, \quad b \leq R_c \]

\[ -2v \ln(k_{pN}b), \quad b > R_c \]

where \( k_{pN} \) is the momentum of the incident proton \( v = Ze^2/\hbar v \), \( Z \) is the charge number of the target nucleus and \( v \) is the incident speed.

2.2. The nuclear phase shift function

The Glauber theory gives

\[ 1 - \exp[i\chi_N(b)] = \langle \psi_0 | \Gamma(b, s_1, s_2, ..., s_N) | \psi_0 \rangle \]  (12)

where \( \psi_0 \) is the ground state of the nucleus, \( \Gamma(b, s_1, s_2, ...) \) is the proton-nucleus profile function and is related to the \( p - \alpha \) profile function by

\[ \Gamma(b, s_1, s_2, ..., s_N) = 1 - \prod_{j=1}^{N} \{1 - I_{p\alpha}(b - s_j)\}. \]  (13)

Table 1. The parameters of the \( P - ^4He \) amplitude at 800 MeV

<table>
<thead>
<tr>
<th>Re ( \beta_1 )</th>
<th>Im ( \beta_1 )</th>
<th>Re ( \beta_2 )</th>
<th>Im ( \beta_2 )</th>
<th>Re ( G_1 )</th>
<th>Im ( G_1 )</th>
<th>Re ( G_3 )</th>
<th>Im ( G_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.424</td>
<td>-0.025</td>
<td>0.490</td>
<td>0.052</td>
<td>-0.330</td>
<td>1.258</td>
<td>0.177</td>
<td>0.295</td>
</tr>
</tbody>
</table>