The Optimal Bankruptcy Rule in a Trading Economy Using Fiat Money*

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1. Introduction

In several previous papers [1, 2, 3, 4, 5, 6, 7] models of a monetary economy have been solved as a noncooperative game. This problem of granting credit and the possibility of bankruptcy was avoided by the artifact of considering that all traders were supplied with "enough" of a commodity serving as a "money" or means of payment so that there was no need to borrow.

In this paper an outside bank, and borrowing are considered explicitly and the meaning of an optimal bankruptcy rule is considered. We stress that if credit or paper money are introduced into an economy described as a game of strategy, rules describing penalties to be levied against those who cannot pay back what they have borrowed, become a logical necessity in order to fully define all possible outcomes. Our approach is to specify such rules and study them parametrically; i. e. we carry out a sensitivity analysis to see what happens as the severity of the penalties is varied.

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This paper deals primarily with problems in modelling and interpretation. Thus the argument is carried out in terms of a specific relatively simple example to illustrate the market, banking and bankruptcy mechanism. General proofs pertaining to a broad class of trading models are given in a separate paper [8].

2. The Model

The model is a variant of the model originally suggested by Shubik [1] and investigated by Shapley [3], Shapley and Shubik [9], Shubik [10, 11] and Dubey and Shubik [5, 6]. The paper here however is self contained inasmuch as a complete model is built, although the references noted provide detailed discussion of some aspects of the model and proofs which are not supplied here.

2.1. A Trading Economy Without Uncertainty

The procedure adopted here is to begin by taking a simple model of trade. This is formulated and solved for the standard competitive equilibrium solution. We then take the same economic background and model trade as a noncooperative game with a bank issuing loans to finance trade.

We solve the game for a type symmetric noncooperative equilibrium point (T. S. N. E.). This is an equilibrium point at which traders of the same type obtain equal treatment. Equal treatment is not necessarily a property of a noncooperative equilibrium. We study the conditions under which the T. S. N. E. coincides or fails to coincide with the C. E. in terms of market prices and distribution of resources.

Consider 2n traders trading in two commodities, n have endowments of (A, 0) and n have endowments of (0, B). Traders of the first type have utility functions of the form

\[ u_1 = \log x_1^1 y_1^{1-\alpha} \]  
\[ u_2 = \log x_2^\beta y_2^{1-\beta}. \]

2.2. The Competitive Equilibrium and Pareto Optimal Surface

It is easy to solve for the unique competitive equilibrium and the Pareto optimal surface. We obtain:

\[ p_1 = 1, \quad p_2 = \left( \frac{1-\alpha}{\beta} \right) \left( \frac{A}{B} \right), \quad x = (1-\alpha) A, \quad y = \beta B, \]

\[ \lambda_1 = 1/A \text{ and } \lambda_2 = \beta/A (1-\alpha), \]