Pairing effects in $^{239}$Pu$(n, 2n)$ reaction cross section

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Abstract. Near-threshold behaviour of $^{239}$Pu$(n, 2n)$ reaction cross section is interpreted within a statistical model. It is shown that an apparent change in the cross section data slope could be attributed to the jump-like excitation of two-quasi-particle states in the residual $^{238}$Pu nucleus. The excitation threshold value is consistent with convenient estimation of the correlation function $A_0$.

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There is a well-known discrepancy between measured data on $^{239}$Pu$(n, 2n)$ reaction cross section [1] and convenient statistical model calculations, which is virtually ignored [2-4]. The data at excitations just above the reaction threshold appear to be much lower than theoretical predictions. In other words, we observe a step-like trend in $(n, 2n)$ data behaviour. Recently we have interpreted step-like behaviour of $^{233}$U neutron-induced fission cross section as due to jump-like excitation of two-quasi-particle states [5]. It occurs in fissioning nucleus $^{235}$U above the pairing gap at saddle deformation. Phenomena of the same nature may manifest themselves in other reaction data also. The $(n, 2n)$ reaction on a $Z$-even, $N$-odd target is one example. Here the behaviour apparent in $(n, 2n)$ cross section data could be correlated with the effect of the same nature occurring in $^{238}$Pu residual nucleus, excited in $^{239}$Pu$(n, 2n)$ reaction. The additional evidence explaining the non-smooth behaviour of that reaction cross section comes from integral data on the $^{239}$Pu$(n, 2n)$ reaction cross section measured with fission spectra neutrons [3].

Statistical description of $(n, 2n)$ reaction cross section

In the vicinity of the $(n, 2n)$ reaction threshold, the competing reactions are $(n, n\gamma)$, $(n, nf)$, and $(n, 2n)$ itself. The main factor defining energy dependence of fission, radiative and neutron emission transmission coefficients is the level density. In the case of the emissive fission reaction $^{239}$Pu$(n, nf)$ it is the level density of $^{239}$Pu at saddle deformations, while for the $(n, 2n)$ reaction it is the level density of $^{238}$Pu at equilibrium deformations. We are dealing here with the second cascade $(n, n \times)$ reaction probabilities, assuming that the ones for first-chance fission $(n, f)$, radiative capture $(n, \gamma)$ and neutron emission $(n, n')$, including the pre-equilibrium contribution, are fixed as described elsewhere [4, 6]. At least they allow crude fitting of the total fission and $(n, 2n)$ data. Some discrepancy of various $(n, 2n)$ reaction cross section calculations [2-4] at incident neutron energies $E=8-20$ MeV could be attributed to different neutron emission/fission competition.

In a near-threshold region one may choose to ignore the data points trend, as the authors of [2, 3] have done, or try to fit the available data [4]. However in both cases they use the smooth level density function, obtained in a Gilbert and Cameron approach [7] as a constant temperature extrapolation of the Fermi gas model [2, 3] or pairing model with an unlimited number of quasi-particles [8], as in [4]. When these approaches are adopted to represent continuum levels of $^{238}$Pu residual nucleus at equilibrium deformations it is impossible to reproduce an apparent step-like behaviour of the data on $(n, 2n)$ reaction.

When the incident neutron energy $E$ is less than $[U_2 + \epsilon_1 + \epsilon_2 + B(239\text{Pu})]$, where $U_2$ is the two-quasi-particle state excitation threshold, $\epsilon_1$ and $\epsilon_2$ are first and second neutron kinetic energies, and $B(239\text{Pu})$ is the neutron binding energy of $^{239}$Pu compound nucleus, the $(n, 2n)$ cross section is governed by collective levels lying within the pairing gap [9]. At higher excitations energies the continuum level density could be represented as follows.

In the adiabatic approximation quasi-particle and collective state contributions to the total level density factorize [8, 10], that is to say

$$\rho(U, J, \pi) = K_{rot}(U, J) K_{vib}(U) \rho_{qp}(U, J, \pi).$$

(1)
Here \( \rho_{qp}(U, J, \pi) \) is the quasi-particle level density at excitation energy \( U \), for angular momentum \( J \), and parity \( \pi \). \( K_{rot}(U, J) \) and \( K_{vib}(U) \) are factors of rotational and vibrational enhancements of level density, respectively. The density of quasi-particle levels \( \rho_{qp}(U, J, \pi) \) could be represented as

\[
\rho_{qp}(U, J, \pi) = \frac{\omega_{qp}(U)}{4\sqrt{2\pi J+1}} \exp\left(-\frac{J(J+1)}{2\sigma^2}\right). \tag{2}
\]

Here \( \omega_{qp}(U) \) is the intrinsic quasi-particle state density, and \( \sigma \) and \( \sigma_1 \) are angular momentum distribution parameters. The angular momentum distribution parameter \( \sigma^2 \) could be represented as

\[
\sigma^2 = \frac{\sum_n \langle m^2 \rangle \omega_n}{\sum_n \omega_n}, \tag{3}
\]

where \( \langle m^2 \rangle = 0.24 A^{2/3} \) is the average value of the squared projection of the angular momentum of the single-particle states; \( \sigma_1 = f_t(t(U)) \), the moment of inertia for rotation with respect to the axis perpendicular to the symmetry axis \( F_2 \); and thermodynamic temperature \( t(U) \) were defined with the pairing model [8]. The \( K_{rot}(U, J) \) value depends in fact on the order of symmetry of the nuclear shape configuration. With actinoid equilibrium deformations the configuration is axially symmetric, so that \( K_{rot}(U, J) \approx \sigma_1^2 \). At saddle deformations of \(^{239}\text{Pu}\) nuclei, fissioning in an \((n, f)\) reaction, at the inner saddle \( K_{rot}(U, J) \approx 2/\pi \sigma_1^2 \sigma \) and at the outer one, \( K_{rot}(U) \approx 2\sigma_1^2 \). The energy dependence of \( K_{vib}(U) \) was defined on the basis of the liquid drop estimate of the nuclear surface multipole vibrations density: \( K_{vib} \approx \exp(U^{2/3}) \) [8].

The \( \omega_{qp}(U) \) could be represented as the sum of \( n \)-quasi-particle state densities \( \omega_n(U) \). The \( \omega_n(U) \) is frequently represented by the Boltzmann gas model expression:

\[
\omega_{qp}(U) = \sum_n \omega_n(U) = \sum_n \frac{g^n(U-U_n)^{n-1}}{(n/2)!^2(n-1)!} \tag{4}
\]

where \( g \) is the single-particle state density at the Fermi surface, \( n \) is the number of quasi-particles, \( U_n \) is the threshold energy for excitation of the \( n \)-quasi-particle configuration, \( n = 2, 4 \ldots \) for even-even nuclei and \( n = 1, 3 \ldots \) for odd nuclei. The \( \omega_n(U) \) is critically dependent on the threshold values for excitation of the \( n \)-quasi-particle configurations. Other authors [11] have shown that the simple formula \( U_n = n \Delta_0, \Delta_0 = 12/\sqrt{A} \), where \( A \) is mass number, overestimates threshold values and distorts an energy dependence of \( \omega_n(U) \) compared with a pairing model with a fixed number of quasi-particles and constant single-particle state density. It has been shown elsewhere [12] that within a convenient Boltzmann gas model the results of the intrinsic state density \( \omega_2(U) \) calculations can be reproduced with an equidistant spectrum pairing model [13]. The threshold values \( U_n \) should be defined as follows from [12]:

\[
U_n = \begin{cases} E_c(3.23n/n_c - 1.57n^2/n_c^2), & \text{if } n \leq 0.446 n_c \\ E_c(1 + 0.627n^2/n_c^2), & \text{if } n > 0.446 n_c \end{cases} \tag{5}
\]

Here, \( n_c = 2 \ln 2 g t_c \), critical temperature \( t_c = 0.571 \Delta_0 \), condensation energy \( E_c = 0.25 g \Delta_0^3 \). This \( U_n \) estimate embodies the energy dependence of a correlation function \( A_n(U) \) as well as a modified Pauli correction to the excitation energy. The shell effects dumping in \( \omega_{qp}(U) \) at high excitations could be modelled with the energy dependence of \( a \)-parameter \( (a = \pi^2 g/6) \):

\[
a(U) = \begin{cases} a(1 + \delta f(U-E_c)/(U-E_c)), & U > 0.47 \Delta_0^2 - m \Delta_0 \\ a_c(U) = a_c, & U \leq 0.47 \Delta_0^2 - m \Delta_0 \end{cases} \tag{6}
\]

Here \( m = 0, 1, 2 \) in case of even-even, odd and odd-odd nuclei, respectively, \( f(x) = 1 - \exp(-0.064x) \), modelling shell effects dumping with energy. Equation (3) corresponds to the case of Fermi particles of one kind, however in [14] it was shown that in the case of Fermi particles of two kinds the energy dependence of \( \omega_n(U) \) (4) changes only slightly. We normalize the \( \rho(U, J, \pi) \) (1–6) by the \( a \)-parameter fitting to the neutron resonance spacing. Note, however, that when modeling a jump-like structure in total intrinsic state density one should keep in mind the rather crude quality of approximation \((\approx 50\%)\) with the Boltzmann gas model \( \omega_2(U) \) of the pairing model \( \omega_2(U) \) [12], especially in the threshold regions of excitation of \( n \)-quasi-particle configurations. The energy dependence of \( \omega_2(U) \) in the Boltzmann gas model, two-quasi-particle excitation threshold \( U_2 \) given by (5) and that in the pairing models [11, 13] are drastically different. In the former case we have \( \omega_2(U) \approx (U-U_2) \), while in the latter \( \omega_2(U) \) is virtually energy-independent above threshold energy \( U_2 \).

The analysis of \(^{239}\text{Pu}(n, 2n)\) reaction data near threshold

The experimental data on the \(^{239}\text{Pu}(n, 2n)\) reaction cross section are shown on the Fig. 1. We observe that the