The branching ratio of $\beta$-delayed two-proton emission

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The decay of the $\beta^+$-fed isobaric analog state by one- and two-proton emission is calculated in the formalism of compound-nucleus decay. Support is given to the uncorrelated character of the emitted protons in the $\beta 2p$ process. The ratio of $\beta 2p$ to $\beta p$ probabilities is obtained for the isotopes predicted to be $\beta 2p$ emitters up to $Z=30$, i.e. $^{22}$Al, $^{22}$Si, $^{26}$P, $^{27}$Si, $^{30}$Ar, $^{34}$Ca, $^{39}$Ti, $^{43}$Cr, $^{46}$Mn, $^{46,47}$Fe, $^{49-51}$Ni, and $^{55}$Zn.

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1. Introduction

The $\beta$-delayed two-proton emission decay mode was predicted some ten years ago by Goldanskii [1]. The first case of this exotic process was observed by Cable et al. [2] in the decay of $^{22}$Al. Up to now, five nuclei have been found to decay by $\beta$-delayed two-proton emission, namely $^{22}$Al [2], $^{26}$P [3], $^{27}$Si [4], $^{31}$Ar [4-7] and $^{35}$Ca [8].

In these five cases, the two protons are emitted from the isobaric analog state (IAS) fed in a super-allowed $\beta^+$ transition from the precursor (Fig. 1). Their emission can proceed through several mechanisms, as represented in Fig. 1, such as sequential emission through an unbound state of the one-proton daughter or direct emission of a correlated or uncorrelated pair of protons. The correlated emission is usually described as a $^2$He emission since the two protons are expected to be in their lowest relative quantum state to maximize the kinetic energy available for the decay. Studies have been made of the kinematic correlation of the two protons emitted in cases where $^2$He emission is allowed by parity conservation. For both $^{22}$Al [9,10] and $^{31}$Ar [11], the $^2$He-emission mechanism was found to contribute very little, if at all, to the $2p$ emission. A recent review [12] summarizes the present status of $\beta 2p$ decay.

The goal of this paper is to discuss one feature which emerges from the scant present experimental information, namely that when $\beta 2p$ decay from the isobaric analog state is observed, it appears to have a rate comparable to that of $\beta p$. This is at variance with the a-priori similar processes of $l p$ and $2p$ (or $l n$ and $2n$) transfer reactions where cross sections of two-nucleon transfers are typically one order of magnitude smaller than those of one-nucleon transfer.

That high rate of $2p$ emission from the isobaric analog state (IAS) can indeed be readily understood if the dominance of the sequential decay mechanism is taken into account. In this paper, the two-proton emission is considered as a one-proton emission to a proton-unbound state of the one-proton daughter (Fig. 1). As a result, the ratio of the rates of the two processes, hereafter noted $\beta 2p/\beta p$, simply reflects the ratio of decays to unbound and bound states of the one-proton daughter nucleus. This in turn is evaluated within the standard formalism used in the analysis of compound-nucleus decay. This formalism rests upon a statistical description of the decay and ignores the detailed spectroscopic properties of the nuclear levels involved. As such it is not expected
to have a strong quantitative predictive value, especially since some of the nuclear levels involved are ground states or low-lying discrete states. However, since the $\beta_2p/\beta p$ ratio exhibits large variations from one isotope to the other, a gross global treatment of the features which determine this ratio appears useful.

In fact, qualitative predictions of $\beta_2p/\beta p$ can be made, simply depending on the values of the three quantities $Q_p$, $Q_{2p}$ (see Fig. 1) and $B_1$ (Coulomb barrier for one-proton emission from the IAS). It is clear that $Q_p \sim Q_{2p}$ will result in a large $\beta_2p/\beta p$ ratio, while on the contrary in a case such as $Q_{2p} < B_1 < Q_p$, a very small $\beta_2p/\beta p$ ratio is expected. Yet a quantitative estimate of these effects is worth calculating, under reasonable assumptions for the level density of the daughter and the proton transmission through the barrier in the emitter.

2. Formalism of the calculation of $\beta_2p/\beta p$ ratio

The decay of the IAS by emission of a proton of energy $E$ is treated in the formalism of the decay of a compound nuclear state [13]. The intensity $I(E)$ of protons of energy $E$ is treated in the formalism of the decay of a compound nuclear state. The second one is the cross section $\sigma$ for the inverse reaction in which a proton of energy $E$ forms the compound nuclear state through the specific channel $C$. The last one, the level density $w$ in the one-proton daughter nucleus $D$ (see Fig. 1), increases strongly with the excitation energy $U = (Q_p - E)$.

As a result, the energy spectrum can be written [13] as:

$$I(E) \, dE = \text{const.} \, E \sigma_c(E) \, w_D(Q_p - E) \, dE.$$  \hspace{1cm} (1)

Since the factor $\sigma_c$ strongly decreases with $E$ at small values of $E$ because of the Coulomb barrier, the function $I(E)$ exhibits a strong maximum, with a typical shape which correctly reproduces the experimental energy spectra of charged particle emission [14]. The same formula also successfully accounts for the shape of $\beta$-delayed proton spectra [15].

The calculation of $I(E)$ according to the above formula rests essentially upon the estimates made for the two main factors, $\sigma_c$ and $w_D$, which are analyzed in the review article of Bodansky [16].

The cross section $\sigma_c(E)$ was first calculated for neutron emission by Feshbach and Weisskopf [17]. The Coulomb barrier hinders the emission of low-energy protons. Values of $\sigma_c(E)$ have been tabulated by Shapiro [18] for $l=0$ proton emission. However, the $l$-value of the emitted proton can be different from 0 depending on the spins and parities of the emitter and daughter states (noted as $J_0^o$ and $J_1^o$ in Fig. 1). The effect of the resulting centrifugal barrier is quite important on the transmission, hence on $\sigma_c(E)$, and has been calculated [19].

The lower the $l$ value, the higher $\sigma_c(E)$. Therefore in this work, $I(E)$ will be calculated with the lowest angular momentum value $l_p$-compatible with the proton emission from the $J_0^o$ IAS to the $J_1^o$ ground state. If $l_p \neq 0$, lower $l$-values will be compatible with proton emission to excited states of the daughter nucleus, i.e. for smaller energies $E$ of the proton. Thus the cross section $\sigma(E)$ will be in most cases a sum over several $l$-dependent partial cross section $\sigma_l(E)$.

The level-density factor $w_D(Q_p - E)$ is discussed in detail by Bodansky [16]. It is usually taken as having the following analytical dependence upon the excitation energy $U = (Q_p - E)$:

$$w(U) = \text{constant} \, U^{-n} \exp \left\{ 2(aU)^{1/2} \right\},$$  \hspace{1cm} (2)

where $a$ is the so-called level-density parameter. Most often $a$ is taken as varying with the mass $A$ of the nucleus as about $A/10$ in MeV$^{-1}$. Yet the most important uncertainty in (2) comes from the exponent $-n$, since values such as $n = 0, 5/4$ and $2$ have been proposed. Their respective merits are discussed by Bodansky [16]. The simplest case, $n = 0$, was used here.

In this and all the following calculations, the mass-excess values of nuclei are taken from Wapstra and Audi [20]. If not available in that compilation, the estimated values are taken from Comay et al. [21]. The excitation energies of the IAS, when not experimentally available, are the calculated values from Antony et al. [22]. The spins and parities are those given by the compilations of Endt an Van der Leun [23] and Lederer and Shirley [24]. In most cases, very little spectroscopic information is available for the proton-rich isotopes involved. So the excited states and $J^o$ values from the better known neutron-rich mirror nuclei were taken without change when needed. For a given $l$-value, $I(E)$ was calculated from (1) by use of the tabulated $\sigma_c$ values of Shapiro [18] corrected for the effect of the centrifugal barrier by the formula and values [19] given by Blatt and Weisskopf [13].

At last (1) is $l$ dependent. When several $l$ transitions can contribute to the energy spectrum for a certain range of $E$ values, a summation on $I(E)$ must be made. In particular, the level-density $w$ depends upon the angular momentum $l$ of the proton since each $l$ transfer corresponds to different $J$ values in the daughter nucleus from the conservation of spin $S = J_o + l + 1/2$. For instance a $l=1$ transition from a $4^+$ IAS may populate all the states with $J^o = 3/2^+, 5/2^-, 7/2^+, 9/2^-$ and $11/2^-$ in the daughter. The $(2J+1)$ dependance [16] of $w$ should then be explicitly introduced in (1). Although this is arithmetically possible, that would force to take into account the detailed spectroscopy of the excited states of the daughter, while in these calculations the emphasis is put on the statistical gross description of nuclear properties. Therefore the $I(E)$ of different $l$-values were added without relative normalization. It was checked in a few cases that no strong distortion of the calculated energy spectrum would result from this simplification.

It must be emphasized again that these calculations only attempt to take into account the gross features of the proton decay which justifies the assumptions made. Accordingly only the large variations of $\beta_2p/\beta p$ from