ANN'S FRACTION SCHEMES

ABSTRACT. A longitudinal constructivist teaching experiment that lasted approximately one academic year was conducted with six third graders. The purpose of the teaching experiment was to analyze the itinerary of children's ways of operating while solving fraction tasks. Ann was one of the third graders who participated in the teaching experiment, and her case study presents the author's interpretation of the generation and evolution of Ann's fraction schemes.

1. INTRODUCTION

The difficulties that children and adolescents experience when operating with fractions have been well-documented by an extensive body of research (among others, Behr, Lesh, Post and Silver, 1983; Behr, Wachsmuth, Post and Lesh, 1984; Behr, Wachsmuth and Post, 1985; Bergeron and Herscovics, 1987; Green, 1969/1970; Hunting, 1980/1981, 1983, 1986; Kerslake, 1986; Kieren, 1980, 1988; Muangnapoe, 1975; and Nik Pa, 1987/1988). The results of the National Assessment of Educational Progress (NAEP) analyzed by Brown, Carpenter, Kouba, Lindquist, Silver and Swafford (1988a, 1988b) indicated that nine-year-olds, thirteen-year-olds, and seventeen-year-olds have low performance in computation with fractions and little conceptual understanding. These findings indicate the students' need to conceptualize fractions as quantities before they are introduced to conventional symbolic algorithms. After all, the algorithms that we use today to operate with fractions are sophisticated procedures generated and refined in the course of the development of mathematics (Waismann, 1959; Smith, vol. 2, 1953; Dantzig, 1954; Courant and Robbins, 1969). In the absence of the conceptual structure behind these algorithms, children memorize them without attached meaning.

For a conventional arithmetical algorithm to become meaningful to a child, it must represent the coordination of mental operations and conventional notations. The premature introduction of conventional notations may represent an obstacle to children's learning of mathematics (Piaget, 1973) by having them use symbols foreign to them to represent their mental operations. Initially, it could be necessary to accept children's informal written representations as transitional notations before conventional notations are negotiated with them. Piaget (1967) views operations with quantities as
mental actions that first take place in experimental contexts under the necessary, but insufficient, influence of language that gives these operations "extension, mobility, and universality" (p. 92). That is, children's use of natural language to verbalize their mental activity could serve as a first step toward symbolization.

Behr et al. (1983, 1985); Kieren (1980, 1988); Kieren, Nelson, and Smith (1985); Piaget, Inhelder, and Szeminska (1960); Peck and Jencks (1981); Pothier and Sawada (1983, 1990); and Streefland (1979, 1984, 1990), among others, have documented the importance of partitioning in the understanding of fractional numbers. McLellan and Dewey (1908) also recognized the importance of "the mental operation of rhythmic parting and wholing" (p. 83) in the generation of units of measurement and fractional units. "Parting and Wholing" seem to be essential mental operations in the development of children's fractional meanings. In the context of natural numbers, the "wholing" operation or unitizing operation has been documented by Steffe (1986, 1988, 1990, 1991) and Steffe and Cobb (1988).

Although the importance of partition and its concomitant emerging units have been suggested in several studies, there have not been studies that focus on the importance of the unitizing operation in the generation of fractional-number units and the relationship between children's ways of operating with natural numbers and their ways of generating fractional numbers and operations with them.

2. METHODOLOGY

2.1. Teaching Experiment

During approximately one academic year, a constructivist teaching experiment was conducted with six third-graders. The objective was to formulate explanations and models of these children's conceptualizations of fractions.

The methodology of the teaching experiment is based on two apparently opposite, but nonetheless complementary, perspectives - the Vygotskian and the Piagetian theoretical frameworks. Under the Vygotskian perspective, social interactions are essential for the internalization of external activity into inner activity. That is, the individual's knowledge is seen as the product of social interaction through which the individual's intellectual growth is determined (Wertsch and Stone, 1985). However, under the Piagetian perspective, social interactions are viewed as a source of cognitive conflict that facilitates the individual's cognitive development. That is, the cognitive subject constructs knowledge by adapting his or her prior knowledge to remain viable in a social or physical world (von Glasersfeld,