VISUAL PROCESSING DURING MATHEMATICAL PROBLEM SOLVING

ABSTRACT. This study investigated, in the context of mathematical problem solving by secondary school students, the nature of the visual schemata which Johnson (1987) hypothesises mediate between logical propositional structures and "rich" specific visual images. Four groups of grade 10 students were studied, representing all combinations of high and low operational ability in mathematics (equivalent to Johnson's logical propositional structures) and high and low vividness of visual imagery (corresponding to Johnson's "rich" images). The results suggested first, that success at problem solving was related to logical operational ability, but not to vividness of visual imagery. Second, a variety of visually based strategies were used during problem solving which differed in their level of generality and abstraction, and use of these strategies appeared related to either logical operational ability or vividness of visual imagery, depending on their level of abstraction. The results supported Presmeg's (1992b) continuum of abstraction of image schemata.

INTRODUCTION

There is increasing recognition that intuitive and imagery based processes play an important role in all levels of mathematical problem solving from those of the child in the early stages of mathematical development (Moses, 1982; Maher and Alston, 1989; Wheatley, 1991); through the non-conventional workplace mathematics of the adult with little formal mathematical training (Resnick, 1987; Scribner, 1984); to the novel creations of highly gifted mathematicians and scientists (Hadamard, 1954; Shepard, 1988). There is, moreover, a more general recognition of a relationship between imagery ability, and problem solving and creativity in areas outside of mathematics (Shaw and DeMers, 1986; Armbruster, 1989; Kaufmann and Helstrup, 1985).

These findings suggest that the development of mathematically competent and creative students involves not only the traditional focus on computational and logical problem solving abilities, but also the development of associated visual imagery and intuitive skills at all stages in the developmental process. Davis (1984, 1986) for instance, argues against overly abstract curricula, maintaining that links between intuitive thought and abstract mathematical symbolisations are frequently not made, resulting in students passing abstract mathematical courses without acquiring "an intuitive 'gut feeling' for the subject" (Davis, 1984: p. 373).
Recognition of the importance of intuitive and imagery based processes in all levels of mathematical problem solving points to the need for a theoretical formulation which accounts for the development of such processes, their role in the construction of meaning and their interaction with logical reasoning abilities. Johnson (1987) has made an important contribution to this field. He differentiates between abstract propositional structures, image schemata, and particular concrete images—"rich" images. He argues that abstract propositions are nonvisual and specify logical rules or abstract relations between symbols and objective reality. One such example would be the concept of a triangle defined as a three sided, closed plane figure (Johnson, 1987: p. 155). In contrast a "rich" image is a specific "picture in the mind", for example of a particular triangle. Mediating between these two forms of thought are image schemata. These operate at a level of generality between abstract propositional structures and specific mental pictures. They are both rule following, partaking of the logical nature of their associated propositions, and they have a sensory component, relating in an abstract, generalised way to specific images and sensations. An image schema of a triangle thus contains visual information not available in the associated abstract concept, but it can be distinguished from the specifics of the visual image of a particular triangle. It can, however, be used to generate the image of any specific triangle.

Image schemata thus form a bridge between abstract logical structures and particular concrete images and experiences. As such, Johnson argues, they play a central role in determining meaning and constraining understanding, a role which is universal in human cognition, but largely unconscious or unrecognised. Johnson also argues that image schemata provide the basis for imagination and account for apparently unbidden intuitions and novel, creative processes.

Presmeg (1992, a&b) has commented on the importance of Johnson's theory for studies of mathematical thinking. In her earlier work (Presmeg, 1986b) she used a mathematical preference for imagery test to distinguish between visualisers who prefer to use visual methods to solve mathematical problems, and nonvisualisers who prefer not to use such methods when problems could be solved by either visual or nonvisual means. This test placed in a single category the use of abstract forms of imagery which Presmeg has termed "pattern imagery", and the more concrete and specific "picture in the mind" types of imagery. Johnson's theory demonstrates the need to clarify these definitions at several levels. First, as Presmeg (1992a) points out, if Johnson is correct in his hypothesis that image schemata are central to the construction of meaning and understanding, then it is inappropriate to divide individuals into visualisers and nonvisualisers, since all