Consider the following variation on the concept of omniscience, where in describing a being as omniscient I mean, as usual,\textsuperscript{1} that whatever is the case, the being knows to be the case: a pair of beings, \(a\) and \(b\) are \textit{collectively omniscient} when whatever is the case is known to be the case by one or other of \(a, b\). Then if each of \(a, b\) is assumed to know all the logical consequences of what he knows, the further assumption that \(a\) and \(b\) are collectively omniscient leads, somewhat surprisingly perhaps, to the conclusion that at least one of them is omniscient \textit{tout court}. For suppose that \(a\) is not omniscient, so that, to use some obvious notation, for some statement \(p\)

\[(i)\quad p \& \neg K_a p\]

and that \(b\) is not omniscient either, so that for some \(q\)

\[(ii)\quad q \& \neg K_b q\]

Then, conjoining the first conjuncts of (i), (ii), we have that the antecedent of (iii) is true:

\[(iii)\quad (p \& q) \rightarrow (K_a (p \& q) \lor K_b (p \& q))\]

a special case of the assumption of collective omniscience for \(a\) and \(b\). But the consequent is false, since its first disjunct conflicts with the second conjunct of (i) and its second disjunct with the second conjunct of (ii). It is in drawing out these conflicts that we make use of the assumption that each of our knowers knows the logical consequences of whatever he knows. This assumption is what epistemologists sometimes call the closure assumption for knowledge, though here it is helpful to make a distinction as modal logicians have done, between a stronger and a weaker version of the idea that knowledge is closed under logical consequences. In general, one describes an operator \(O\) (which forms a sentence when attached to a sentence) as \textit{monotonic} (or 'monotone') when \(OB\) is a consequence of \(OA\) whenever \(B\) is a consequence

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of \( A \), for any \( A \) and \( B \), and as regular when \( OB \) is a consequence of \( OA_1, \ldots, OA_n \) \( (n \geq 1) \) whenever \( B \) is a consequence of \( A_1, \ldots, A_n \), for any \( A_1, \ldots, A_n, B \); evidently, though the converse does not hold, any regular operator is monotonic (take \( n = 1 \)), so that regularity for the operators \( K_a, K_b \) is the stronger closure assumption, while all that is required for the above argument is monotonicity.\(^2\) It is to be observed, further, that though even this weaker assumption is sometimes held to be an absurdly strong demand on the deductive powers of the normal subjects of knowledge-attributions, the only extent to which it is exploited in the above argument is inferring knowledge (on the part of \( a \) and \( b \)) of a conjunct from knowledge of a conjunction containing that conjunct, and the degree to which this appears an idealization is surely negligible.

The argument called attention to here can be recast in the bimodal logic of the two monotonic operators \( K_a, K_b \), as a deduction of \( '(p \rightarrow K_a p) \lor (q \rightarrow K_b q)' \) from \( 'p \rightarrow (K_a p \lor K_b p)' \), thus:

\[
\begin{align*}
(1) & \quad p \rightarrow (K_a p \lor K_b p) \\
(2) & \quad (p \& q) \rightarrow K_a(p \& q) \lor K_b(p \& q)) \quad \text{From 1, substituting } p \& q \text{ for } p \\
(3) & \quad K_a(p \& q) \rightarrow K_a p \quad \text{By Monotonicity applied to } (p \& q) \rightarrow p \\
(4) & \quad K_b(p \& q) \rightarrow K_b q \quad \text{By Monotonicity applied to } (p \& q) \rightarrow q \\
(5) & \quad (p \& q) \rightarrow (K_a p \lor K_b q) \quad \text{A Truth-functional consequence of 2, 3, 4} \\
(6) & \quad (p \rightarrow K_a p) \lor (q \rightarrow K_b q) \quad \text{Truth-functionally equivalent to 5}
\end{align*}
\]

It is to be noted that neither of the disjuncts of (6) can be deduced from (1), only their disjunction; and in my opening remarks I said if \( a \) and \( b \) were collectively omniscient it followed that one or other of them was omniscient, not that one or other of them was such that his omniscience followed from the collective omniscience of the pair. Thus if we think of (1) as an axiom extending the smallest bimodal monotonic logic to give a system we might consider in its own right, we see that this system is, as they say, Hallden-incomplete (or Hallden-unreasonable): it contains disjunctive theorems without containing either disjunct as a theorem even when the disjuncts in question have no propositional variable in common. I have been careful here to