ON STABLE AND UNIFORM RANK-2 VECTOR BUNDLES ON P² IN CHARACTERISTIC p

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The uniform rank-2 vector bundles on P^n are determined and the behaviour of the stable rank-2 vector bundles on P² under restriction to a general line is studied, where P^n denotes the n-dimensional projective space over an algebraically closed field of positive characteristic.

Introduction: In the theory of rank-2 vector bundles on P^n in characteristic 0 there are 2 important theorems, the proofs of which use the characteristic 0 assumption, namely the theorem of van de Ven, which says that a uniform rank-2 vector bundle on P^n is either a direct sum of line bundles or a twist of the tangent bundle on P² (cp. [6]) and the theorem of Grauert-Mülich, which says that the stability-degree of the restriction of a stable rank-2 vector bundle to a general line is 0 or -1 (cp. [1]). (For the definitions compare section 2).

The first question is: are the characteristic restrictions necessary? It is easy to see that they are. Hence the problem arises: What is the situation in characteristic p>0? The aim of this paper is to determine the uniform rank-2 vector bundles on P^n and to prove the corresponding result of the theorem of Grauert-Mülich on P², both for characteristic p>0. It turns out that there are more uniform rank-2 vector bundles (cp. Theorem 2.4) and that the above mentioned stability-degree can become arbitrarily negative (cp. Theorem 3.1).
The proofs of both theorems use the original proofs whenever possible. Especially for sake of shortness the "standard construction" of [1],4 is not repeated.

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1) This section contains some auxiliary results on the Frobenius morphism used in sections 2 and 3. Let k be an algebraically closed field of characteristic p>0. If f: X → Y is a morphism of algebraic varieties over k, let $F_Y$ denote the relative Frobenius morphism on X that is defined by the following diagram, where F means the absolute Frobenius

![Diagram of Frobenius morphism]

If Y is the spectrum of a field $\kappa$, we write $F_\kappa$ instead of $F_{\text{Spec}(\kappa)}$. If $\kappa=k$, then X and $Y_{X,Y}$ are canonically isomorphic only having a different $\kappa$ structure and $F_\kappa$ is just the $k$-linear Frobenius.

Now let $F^n$ be projective n-space over k and E a rank-2 vector bundle on $F^n$, that is a locally free $O_{F^n}$-module of rank 2.

Then we have for any line $l$ in $F^n$:

**Lemma 1.1:** If $E/l = O_{F^n}(a) \oplus O_{F^n}(b)$ then $F^*E/l = O_{F^n}(p.a) \oplus O_{F^n}(p.b)$.