Effects of Electron-Hole Correlation on Ultrasonic Attenuation in Bismuth in Strong Magnetic Fields

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A theory is developed on giant quantum attenuation of ultrasound in bismuth. The present theory successfully explains the following experimental results in strong magnetic fields ($H \leq 100$ kG): (i) When two attenuation peaks, the one due to electrons and the other due to holes, coincide as a function of magnetic field, the attenuation is exceptionally large at temperatures around 1 K and decreases rapidly with increasing temperatures; (ii) on the contrary, an isolated attenuation peak shows only a weak temperature dependence; (iii) the line shape of an isolated hole peak is highly asymmetric. The theory includes both intraband and interband impurity scatterings, acoustic phonon scattering, and takes account of Coulomb correlation effects via electron-electron, hole-hole and electron-hole two-body distribution functions. As a result, the electron-hole attractive correlation is found to play a crucial role in making the large attenuation mentioned in (i). For (ii), the electron-hole correlation is ineffective because of the large difference in Fermi velocities, and the acoustic phonon scattering is found to be important. Finally, the result (iii) is attributed to the small density of states of the reservoir Landau subbands in the strong magnetic field regime. The present theory assumes no phase transition to account for the result (i) in contrast to previous theories.

1. Introduction

The attenuation of ultrasound in clean metals and semimetals shows a series of spike-like peaks as a function of magnetic field at low enough temperatures. It is well established that the spike-like peak, which is called giant quantum attenuation (GQA), occurs when the Fermi level comes close to the bottom of one of the Landau subbands. Thus investigation of the period of GQA provides information about the Fermi surface of the material, just as in the case of de Haas-van Alphen and Schubnikov-de Haas effects. However it should be noted that the ultrasound can probe much richer aspects of the system because of its dynamical ($\omega \neq 0$) and spatially dispersive ($\mathbf{q} \neq 0$) nature. The richness becomes clear by the following, apparently puzzling experimental results [1] obtained for bismuth semimetal in strong magnetic fields ($H \leq 100$ kG): (i) when two attenuation peaks, the one due to electrons and the other due to holes coincide, the attenuation becomes exceptionally large ($\approx 10$ dB/cm for 100 MHz ultrasound) at about $T = 1$ K and decreases rapidly with increasing temperatures; (ii) on the contrary, isolated attenuation peaks show only a weak temperature dependence; (iii) the line shape of the isolated hole peak is highly asymmetric. Neither the original theory of GQA by Gurevich et al. [2] nor a number of revised theories [3, 4] can explain these features.

In the present paper we put forward a unified theory of GQA that can account for all of the experimental features mentioned above. First of all we note that at least three kinds of scattering mechanisms are known to be important in transport phenomena in bismuth at liquid helium temperatures: (a) intraband impurity scattering [5]; (b) interband impurity scattering [6, 7]; (c) acoustic phonon scattering [8]. In addition, the smallness of the kinetic energy of electrons which take part in the ultrasonic attenuation process...
implies that the Coulomb interaction may also be important. In this paper we present a simple theoretical scheme which can incorporate all of these effects in a tractable manner. We adopt the self-consistent Born approximation for the scattering processes (a)–(c) and a generalized version of the random phase approximation (RPA) formulated by Singwi et al. [9] for the Coulomb interaction.

In the field of about 100 kG, bismuth still has a few Landau subbands with large Fermi velocities. Electrons in these subbands act as a screening medium in the attenuation process with the Thomas-Fermi screening parameter \( \kappa \). A numerical estimate shows that \( k_0 \approx \kappa < l^{-1} \), where \( k_0 \) is a characteristic wave number of electrons or holes giving rise to GQA, and \( l = (c/eH)^{1/2} \) is the measure of the extension of wave functions perpendicular to the magnetic field. Thus the screened Coulomb interaction is regarded as quite short-ranged in the direction parallel to the field, but should not be weak because the averaged interparticle distance perpendicular to the field is about \( l \). The strong anisotropy permits us to account for the Coulomb interaction through the two-body distribution functions \( g_{ij}(r) \) with \( r \approx 0 \), where \( i \) and \( j \) refer to either electron or hole. In the present approach we can properly take into account the incoherent exciton-like correlation between electrons and holes, the importance of which has been suggested by Kuramoto and Morimoto [10]. We should remark that the perturbation theoretic approach adopted by previous work [11–13] is not suitable to account for the actual correlation in bismuth which in fact has a very short correlation length.

Although it requires a considerable amount of numerical calculation to obtain the attenuation coefficient, the mechanisms that give the experimental properties (i)–(iii) permit a simple physical interpretation. This discussion is given in Sect. 5 after detailed results of the numerical calculation and the comparison with experiments are presented in Sect. 4.

In Sect. 2, the ultrasonic attenuation formula is given in terms of the density response functions and the scheme to account for the Coulomb interaction is developed. In the generalized RPA, the density response functions without the Coulomb interaction must first be obtained. They are calculated in Sect. 3 with impurity and acoustic phonon scatterings included. Section 6 is devoted to concluding remarks. Throughout the paper we use the units \( \hbar = k_B = 1 \).

### 2. Attenuation Formula in the Generalized RPA

In this section we first derive ultrasonic attenuation formula in terms of density response functions for both conduction and valence electrons. Then the formalism to take into account the correlation effect is described.

#### 2.1. Attenuation Formula

In bismuth, the most important interaction between ultrasound and carriers is determined by the deformation potential tensor \( A_{ij}^{\alpha} \) for the \( i \)-th electron pocket and \( A_{ij}^{\alpha} \) for the hole pocket, where \( \alpha \beta \) are tensor indices [14]. The interaction Hamiltonian is given by

\[
\mathcal{H}_{im} = \int d\mathbf{r} \sum_{\alpha\beta} \varepsilon_{\alpha\beta}(r,t) \cdot \left[ \sum_{i=1}^{3} A_{ij}^{\alpha\beta} \rho_i(r,t) + A_{ij}^{\alpha\beta} \rho_v(r,t) \right].
\]  

(2.1)

where \( \varepsilon_{\alpha\beta} \) is the strain tensor, \( \rho_i \) and \( \rho_v \) are the density operators of the \( i \)-th pocket conduction electron and of the valence electron, respectively. In this paper the discussion is confined to the case [1] where longitudinal ultrasound is propagated along the trigonal axis of the crystal \( (\alpha = \beta = 3) \), and therefore \( A_{ij}^{33} \) is independent of \( i \). Then the linear response theory yields the following expression for the ultrasonic attenuation coefficient \( \eta \) [15]:

\[
\eta = \frac{1}{2} \left| \frac{q}{\rho_m v_s^2} \right| \text{Im} \left\{ A_{\alpha}^{22} R_{-1}(q, \omega) + A_{\alpha}^{22} R_{+2}(q, \omega) \right. \\
+ \left. A_{\alpha}^{22} A_{\alpha}^{22} R_{-1}(q, \omega) + R_{+2}(q, \omega) \right]\}
\]  

(2.2)

where \( q \) and \( \omega \) are the wave number and frequency of the ultrasound with velocity \( v_s(\omega = v_s |q|) \), \( \rho_m \) is the mass density of bismuth \((=9.8 \text{ g cm}^{-3})\), and

\[
A_{\alpha}^{22} = 2^{-1} \left( A_{\alpha}^{33} \pm A_{\beta}^{33} \right)
\]  

(2.3)

\[
R_{\pm}(q, \omega) = iL^{-3} \int_0^\infty \text{d}t e^{i\omega t} \text{c}_{\text{stat}} \langle [\rho_{\pm}(q, t), \rho_{\pm}(-q, 0)] \rangle
\]  

(2.4)

with

\[
\rho_{\pm} = 2^{-1/2} (\rho_e \pm \rho_h), \quad \left( \rho_e = \sum_{i=1}^{3} \rho_{ei} \right)
\]  

(2.5)

and \( L \) being the dimension of the system. In (2.4), \([\ldots, \ldots] \) means commutator and \( \langle \ldots \rangle \) the statistical average.

The motivation of writing (2.2) in terms of \( \rho_{\pm} \) operators instead of \( \rho_c \) and \( \rho_v \) is that only the response function \( R_{\pm}(q, \omega) \) remains to be considered. Other response functions involve charge density \( \rho_+ \) response, so that a strong screening effect makes them negligibly small.