Influence of the soliton-magnon interferences on the dynamics of an antiferromagnetic chain

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We study the influence of small amplitude fluctuations on the soliton induced dynamic structure factors of the classical one dimensional antiferromagnet with two single ion anisotropies. In contrast to the case of an external magnetic field the equations of motion for the in-plane and out-of-plane fluctuations separate and can be solved exactly.

1. Introduction

In the past few years the concept of solitons has been successfully applied to magnetic chains [1]. The one-dimensional antiferromagnetic compound (CD₃)₄NMnCl₄(TMMC) in a transverse field has been shown to exhibit dramatic effects associated with the presence of solitons [2, 3]. Although the thermodynamical properties of non linear excitations are, by now, quite well understood, in many respects there are still some open questions in the dynamics. In this paper we present theoretical results on the dynamic structure factors of the classical antiferromagnetic chain. We will be concerned with the interference effects between solitons and magnons, following the method proposed by Allroth and Mikeska to treat the sine-Gordon like magnetic chain [4]. Since mathematical difficulties precludes a full treatment of the antiferromagnet in an external magnetic field [5] we will consider the case of an antiferromagnet with two anisotropies described by the Hamiltonian

$$H = 2J \sum_{n} [S_n \cdot S_{n+1} + \delta(S_n^2) + b(S_n^2)].$$

The classical ground state configuration of (1) for $\delta, b > 0$, is Ising like, $S = (\mp S, 0, 0)$. Here we make use of the angle variables introduced by Mikeska [6] to treat one-dimensional classical antiferromagnets:

$$S_n = (-1)^n S \{ \cos(\theta_n + (-1)^n \phi_n)$$
$$\cdot \cos(\phi_n + (-1)^n \psi_n), \cos(\theta_n + (-1)^n \phi_n)$$
$$\cdot \sin(\phi_n + (-1)^n \psi_n), \sin(\theta_n + (-1)^n \phi_n) \},$$

$\theta$ and $\phi$ are angles giving the sublattice magnetization, whereas $\psi$ and $\zeta$ describe the deviations from perfect anti-alignment, and can be assumed to be small at low temperatures. In the continuum approximation Hamiltonian (1) becomes [7]

$$H = JS \int dz \left\{ \frac{1}{c^2} \left[ \left( \frac{\partial \theta}{\partial t} \right)^2 + \cos^2 \theta \left( \frac{\partial \phi}{\partial t} \right)^2 \right] \right. \right.$$

$$+ \left( \frac{\partial \theta}{\partial z} \right)^2 + \cos^2 \theta \left( \frac{\partial \phi}{\partial z} \right)^2$$
$$\left. + 2 \delta \sin^2 \theta + 2 b \cos^2 \theta \sin^2 \phi \right\}.$$

where $C = 4JS$, and $z$ is the coordinate along the chain.

The equation of motion can be obtained directly from Hamiltonian (3) and are presented in [7], where it is also shown that we have two soliton solutions, one in the $xz$ plane and the other in the $xy$ plane. For $\delta > b$ the $XY$ soliton has lower energy and is described by

$$\theta = 0, \quad \phi = 2 \tan^{-1} [\exp(\sqrt{2b(z-ut)})],$$

$$\nu = -(\sqrt{2b}u/4JS) \sech [\sqrt{2b(z-ut)}], \quad \zeta = 0,$$

with energy

$$E_{xy} = E_{xy}^{0}, \quad E_{x} = 4JS^2 \sqrt{2b},$$

where $\gamma = (1-u^2/C^2)^{-1/2}$. 
2. Soliton-magnon interference effects

In this section we consider the modification of the magnon solutions due to the presence of a $X Y$ soliton. The behavior of small oscillations in the presence of a single static soliton $\phi_{\text{sol}}(z)$ is determined by solutions of the form

$$\theta(z, t) = \bar{\theta}(z, t), \quad \phi(z, t) = \phi_{\text{sol}}(z) + \bar{\phi}(z, t).$$

Substitution of (6) in the equation of motion, linearization in $\theta$ and $\phi$ and writing $\theta$ and $\phi$ as

$$\bar{\theta}(z, t) = s(z) e^{i \omega_1 t}, \quad \bar{\phi}(z, t) = r(z) e^{i \omega_2 t}$$

leads to the following eigenvalue equations:

$$\frac{d^2 r}{dz^2} - 2b(1 - 2 \text{sech}^2 \sqrt{2b} z) r = -\frac{\omega_1^2}{C^2} r,$$

$$\frac{d^2 s}{dz^2} - 2b(1 - 2 \text{sech}^2 \sqrt{2b} z) s = -\frac{\omega_2^2}{C^2} s,$$

where

$$\omega_1 = \omega_1^2 - 2(\delta - b) C^2.$$ 

The dispersion relation is determined by the behavior far from the soliton centre. We find

$$\omega_1^2 = 2(\delta + q^2) C^2, \quad \omega_2^2 = 2(2b + q^2) C^2.$$ 

The scattering solutions of (8) and (9) are [8]

$$\tilde{\theta}_q(z, t) = e^{i(qz - \omega_1 t)} A_{\theta q}(z, t) e^{i \xi_{\theta q}(z, t)},$$

$$\tilde{\phi}_q(z, t) = e^{i(qz - \omega_2 t)} A_{\phi q}(z, t) e^{i \xi_{\phi q}(z, t)},$$

where in the nonrelativistic limit

$$A_{\theta q}(z, t) = \left\{ 1 - m^2 \frac{\text{sech}^2[m(z - u_n t - z_{0a})]}{q^2 + m^2} \right\}^{1/2},$$

$$\xi_{\theta q}(z, t) = \tan^{-1} \left[ \frac{m}{q} \text{tanh}[m(z - u_n t - z_{0a})] \right] - \tan^{-1} \left[ \frac{m}{q} \text{tanh}[m z_{0a}] \right],$$

where $m = \sqrt{2b}$.

At low temperatures the generalisations of the solutions (12) and (13) to the case of $N$ solitons and $N$ antisolitons can be written as

$$\tilde{\theta}_q(z, t) = e^{i(qz - \omega_1 t)} \prod_{n=1} A_{\theta q}(z, t) e^{i \xi_{\theta q}(z, t)},$$

$$\tilde{\phi}_q(z, t) = e^{i(qz - \omega_2 t)} \prod_{n=1} A_{\phi q}(z, t) e^{i \xi_{\phi q}(z, t)}.$$ 

For periodic boundary conditions the density $\rho(q)$ of magnons states in $q$-space for a chain of length $L \to \infty$ is

$$\rho(q) = \frac{L}{2\pi} \left\{ 1 - \frac{4nm}{m^2 + q^2} \right\},$$

where $n$, the soliton density, is given by [7]

$$n = n_{\text{sol}}(1 + \sqrt{b/\delta})(1 - b/\delta)^{-1/2},$$

$n_{\text{sol}}$ being the soliton density for the sine-Gordon model

$$n_{\text{sol}} = 4\sqrt{2/\pi} J^{1/2} S m (m/T)^{1/2} \exp(-4JS^2 m/T).$$

For an study of the influence of the soliton-magnon interferences on the zero order magnon and soliton peaks we consider a linear superposition of $2N$ (anti)solitons and the real part of a general magnon solution [4]:

$$\theta(z, t) = \text{Re} \sum_q \tilde{\theta}_q(z, t),$$

$$\phi(z, t) = \sum_{n=1}^{N} \phi_{\text{sol}}(z - u_n t - z_{0a}) + \text{Re} \sum_q \tilde{\phi}_q(z, t).$$

2.1. The longitudinal structure factor

The longitudinal dynamic structure factor is given by

$$S_{xx}(q, \omega) = \int d t d z e^{i(qz - \omega t)} \left( g_{xx}(z, t) + h_{xx}(z, t) \right),$$

where for the $XY$ soliton and to lowest order in $v,$

$$g_{xx}(z, t) = (-1)^{n} \cos \theta(z, t) \cos \theta(0, 0) \cdot \cos \phi(z, t) \cos \phi(0, 0),$$

$$h_{xx}(z, t) = -(-1)^{n} \cos \theta(z, t) \cos \theta(0, 0) \cdot \cos \phi(z, t) \cos \phi(0, 0),$$

and $z = na$, $a$ being the lattice parameter. Following the same steps as in [4] we find, to lowest order in the soliton density

$$g_{xx}(z, t) = g_{0x}^{xx}(z, t) - \frac{1}{2} \sum_{k} \left[ \langle | \lambda_k |^2 \rangle + \langle | \gamma_k |^2 \rangle \right] (2k^2(z, t))^{2N} - 2N \left[ 1 - \cos \phi_{\text{sol}}(z, t) \cos \phi_{\text{sol}}(0, 0) \right] \cdot \left[ A_{\phi q}(z, t) + A_{\phi q}(0, 0) \right] \langle | \lambda_k |^2 \rangle$$

$$- \sum_{k} \left[ 2N \text{Re} e^{i(kz - \omega_2 t)} \cdot \left( \sin \phi_{\text{sol}}(z, t) \sin \phi_{\text{sol}}(0, 0) \cdot A_{\phi q}(z, t) - A_{\phi q}(0, 0) \cdot A_{\phi q}(z, t) \right) \right].$$

(26)