A Semi-Phenomenological Nuclear Optical-Model Potential

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A new semi-phenomenological form of the nuclear optical-model potential for the description of the average elastic scattering of nucleons is proposed. It consists of an energy-independent, nonlocal “single-particle” term and a sum of energy-dependent, factorable terms exhibiting “intermediate structure”.

1. Introduction

In a recent paper¹, a form of the generalized optical-model potential for elastic scattering was proposed which yields the unaveraged elastic scattering amplitude with full account of the narrow compound nucleus resonances.

The aim of the present paper is to derive from the results of ref. ¹ an optical-model potential in the proper sense, which describes the elastic scattering amplitude averaged over some energy interval.

In sect. 2 the form of the generalized optical-model potential of ref. ¹ is further specified. In sect. 3 the procedure of taking the energy average is explained. It is found that the proper optical-model potential is approximately equal to the energy average of the generalized optical-model potential. It is therefore also “causal” and satisfies a dispersion relation. The proper optical-model potential obtained in this way is further simplified in sect. 4. An attempt is made to reduce it to an intuitively appealing semi-phenomenological form with a limited number of parameters, so that it can be used in the numerical analysis of experimental data.

Sect. 5 contains some concluding remarks.

2. The Generalized Optical-Model Potential

The generalized optical-model potential proposed in ref. ¹ has the following form (Eq. (6.43) of ref. ¹):

$$\mathcal{V}(x, x') = \mathcal{V}^{(0)}(x, x') + \mathcal{V}^{(direct)}_{E}(x, x') + \sum_{i} V_{iE}(x) V_{iE}(x') G_{i}(E).$$  \hspace{1cm} (2.1)

Here $\mathcal{V}^{(0)}(x, x')$ is an energy-independent term representing the average interaction of the extra particle with the nucleons in the target nucleus

in the ground state (shell-model or Hartree-Fock potential). The sum of factorable terms arises from the coupling of the continuum to discrete bound or quasi-bound states below and in the continuum, labeled by the index \( i \). The quantities \( V_{iE}(x) \) characterize the strength of this coupling, and the coordinate-independent factors

\[ G_i(E) = \left[ E - E_i - \int \frac{u_i(E') dE'}{E - E' + i \eta} \right]^{-1} \tag{2.2} \]

are weight functions with poles on the real \( E \)-axis for \( E < \varepsilon_1 \) (\( \varepsilon_1 \) is the lowest inelastic threshold) or below the real axis on unphysical sheets for \( \text{Re} \, E > \varepsilon_1 \). The sum in Eq. (2.1) contains many terms with strongly fluctuating weight functions \( G_i(E) \), corresponding to the many narrow resonances in the elastic scattering cross section. Finally, the term \( \psi^{(\text{direct})}(x, x') \) represents the contribution from the direct coupling between elastic and inelastic continuum channels without the intermediary of the discrete states. It is expected to be small and weakly energy-dependent if a shell-model description is to be meaningful at all.

In keeping with our expectation that the direct coupling between the continua is not very important compared with the coupling to the discrete states, we may regard the quantities \( V_{iE}(x) \) as real and energy-independent. This is seen from the shell-model calculation in ref. \(^1\). Neglecting in (6.40) of ref. \(^1\) the second term on the right-hand side due to the direct coupling, we are left with the term \( V_{ik}^{(0)} \), which becomes in the coordinate representation (cf. (A.20) of ref. \(^1\))

\[ V_i(x) = \sqrt{A + 1} \langle \Phi_i(0, \ldots, A), V g(1, \ldots, A) \rangle, \tag{2.3} \]

where \( \Phi_i \) is the wave function of the discrete state, \( g \) is the wave function of the target nucleus, and \( V \) is the residual interaction,

\[ V = \sum_{i<j}^A v_{ij} - \sum_{i=0}^A V_0(i) \tag{2.4} \]

(for further details see ref. \(^1\)).

The shell-model interpretation of the scattering process can be carried further, following MAHAUX and WEIDENMULLER \(^2\). This leads to further insight in the structure of the quantity \( V_i(x) \). The energies \( E_i \) in Eq. (2.2) are the eigenvalues of the total Hamiltonian in the space of the

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