On the Description of Low Energy Nuclear Resonances with the Help of Quantum Green's Functions

E. WERNER
Institut für Theoretische Physik der Technischen Hochschule Hannover

Received September 7, 1966

It is shown how the appearance of low energy nuclear resonances in the elastic channel can be described with the help of quantum Green's functions. After the development of the general theory a model is sketched which can be used to perform explicit calculations.

1. Introduction

Recently considerable progress has been made in the understanding of low-energy nuclear resonances. In this context one should mention the introduction of the idea of door-way states\textsuperscript{1} and WEIDENMÜLLER's work on shell-model calculations in the continuum\textsuperscript{2}. On the other hand the general understanding of low-energy nuclear phenomena has much been augmented by MIGDAL's papers on the quasi-particle theory of nuclear structure\textsuperscript{3}.

In this paper we give a description of low-lying resonances in the elastic channel with the help of quantum Green's functions. In a sense the proposed method constitutes an extension of the work of WEIDENMÜLLER; it avoids the explicit use of wave functions and can therefore be used in rather general situations.

Our purpose is the evaluation of the $T$-matrix for the elastic scattering of a nucleon of energy $E$ by a nucleus. The $T$-matrix can be written as\textsuperscript{4}

$$
T = M + MG_0 T.
$$

(1)

Here $G_0$ is the free single-particle propagator and $M$ is DYSON's self-energy operator defined by the equation

$$
G = G_0 + G_0 MG.
$$

(2)

$G$ being the exact single-particle propagator. Eq. (1) can be written in another form which is more advantageous for our purposes. From


Description of Low Energy Nuclear Resonances

Eq. (1) we have

\[(1 - M G_0) T = M.\]

From Eq. (2) we obtain by successive multiplication with \(G_0^{-1}\) from the left and \(G^{-1} G_0\) from the right

\[1 - M G_0 = G^{-1} G_0.\]

This gives

\[T = G_0^{-1} G M.\] (3)

Since according to Eq. (2) we have

\[G_0^{-1} G = 1 + M G\]

we obtain finally

\[T = M + M G M.\] (4)

In order to proceed further one has to introduce a single-particle basis. It will be characterized by the quantum numbers \((n, j, m, \pi)\) for discrete states and by \((k, j, m, \pi)\) for continuum states, \(n\) and \(k\) being the radial quantum and the asymptotic wave number respectively.

We want to assume spherical symmetry so that \(M\) and \(G\) are automatically diagonal in \(j, m\) and \(\pi\). We can therefore suppress the quantum numbers \(j, m, \pi\) and it will be understood that all formulas refer to the scattering of nucleons having definite values of \(j, m\) and \(\pi\) relative to the target nucleus.

2. The Equations Determining the Quantities \(M\) and \(G\)

We want to treat a situation where the energy of the incident particle is so low that the resonances in the elastic channel are well separated.

In this region the single-particle-propagator \(G_{kk'}(\omega)\) will be of the following form

\[G_{kk'}(\omega) = \frac{z_k \delta_{kk'}}{\omega - \epsilon(k) + i \delta} + \text{sum of pole terms},\] (5)

\[= \delta_{kk'} g_k(\omega) + \text{sum of pole terms},\] (5a)

where

\[\epsilon(k) = \frac{\hbar^2 k^2}{2m},\]

and the sum goes over poles lying in the lower half of the \(\omega\)-plane. In a general situation the residues of several poles can be of comparable size \(^5,6\). Besides these poles there will be a great number of poles with small residues.
