In the classical collision theory the scattering angle \( \theta \) depends on the impact parameter \( b \) and on the kinetic energy \( E_r \) of the relative motion. This angle \( \theta(b, E_r) \) is expanded for two limiting cases: 1. Expansion in powers of the potential \( V(r)/E_r \) (momentum approximation). 2. Expansion in powers of the impact parameter \( b \) (central collision approximation).

The radius of convergence of the series depends on \( b \) and \( E_r \). It will be given for the following potentials \( V(r) \):

\[
\begin{align*}
A\left(\frac{a}{r}\right)^2; \\
A e^{-\frac{r}{a}}; \\
A \frac{a}{r} e^{-\frac{r}{a}}; \\
A \left(\frac{a}{r}\right)^2 e^{-\left(\frac{r}{a}\right)^2}.
\end{align*}
\]

For case 1 a \( b \)-dependent criterion is deduced for the alternating of the series. Then the error is smaller than the first neglected term.

1. Introduction

In the classical theory of collisions between two particles interacting with a central potential the scattering angle \( \theta \) in the centre of mass system is given by

\[
\theta = -2 \int_{r_m}^{\infty} \frac{b \, d \, r}{r^2} \frac{1}{\left(1 - \frac{V(r)}{E_r} - \frac{b^2}{r^2}\right)^{\frac{1}{2}}}
\]

with

\[
v(r) = \frac{V(r)}{E_r}; \quad 1 - v(r_m) - \frac{b^2}{r_m^2} = 0
\]

here \( V(r) \) is the potential*, depending on the interatomic distance \( r \), \( E_r \) is the kinetic energy of the relative motion \( b \) is the impact parameter and \( r_m(b) \) is the distance of closest approach.

In the integral (1) the path of integration extends from \( r_m \) along the real axis to infinite. This path can be replaced by a path \( C \) (Fig.1) in

\* Only repulsive potentials are considered.

the complex $r$-plane, because of the square root singularity at $r_m$. Then the scattering angle $\theta$ can be expanded for two limiting cases\textsuperscript{2,3}.

Fig. 1. Path of integration in the complex $r$-plane

1. Expansion in powers of the potential (momentum approximation). One obtains

$$\theta = \sum_{n=1}^{\infty} \frac{1}{\gamma_n} \int \frac{b \, dr}{r^2} \frac{v^n(r)}{\left(1 - \frac{b^2}{r^2}\right)^{n+\frac{1}{2}}}.$$ \hspace{1cm} (2)

All integrals have a singularity at $r = b$ independent of $v(r)$.

2. Expansion for small $b$ (central collision approximation). The result is:

$$\theta = \pi - \sum_{n=0}^{\infty} \frac{1}{\gamma_n} \int \frac{b \, dr}{r^2} \frac{1}{\left(1 - v(r)\right)^{n+\frac{1}{2}}}.$$ \hspace{1cm} (3)

In this case the singularity is at $r = r_m(b = 0)$. For both expressions is

$$\gamma_n = (-1)^n \frac{(2n-1)!}{2^{n-1} (n-1)! n!} \leq \frac{1}{\sqrt{\pi n}}.$$  

The inequality holds for $n \geq 2$.

In particular the series (2) and (3) will be treated for the potentials $V(r)$:

$\frac{a}{r}$ \hspace{1cm} (4a); \hspace{1cm} $A e^{-\frac{r}{a}}$ \hspace{1cm} (4b); \hspace{1cm} $A \frac{a}{r} e^{-\frac{r}{a}}$ \hspace{1cm} (4c); \hspace{1cm} $A \left(\frac{a}{r}\right)^2 e^{-(\frac{r}{a})^2}$ \hspace{1cm} (4d).

2. The Convergence Behaviour

The radius of convergence of the power series (2) and (3) depending on $A/E$, and $b$ can be determined by the criterion

$$\lim_{n \to \infty} \left| \frac{\gamma_{n+1}}{\gamma_n} \right| < 1.$$
