Mathematical model of a single-screw plasticating extruder

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Abstract: A five zone mathematical model of a plasticating extruder is presented. Its application in the design of new and improvement of existing extruders is briefly described. The model is based on theories proposed by Darnell and Mol, Tadmor, Broyer, McKelvey, Klein, Schneider, Fenner, Poon and Jankov. A comparison between experiments and theoretical calculations is included.

Key words: Single-screw extruder, mathematical model, polypropylene

Nomenclature

\[ E \] energy, W
\[ f \] melt film thickness, m
\[ f_k \] friction coefficient
\[ h \] channel depth, m
\[ l \] axial screw distance, m
\[ k \] power-law parameter, °C\(^{-1}\)
\[ m_0 \] power-law parameter, Pa \cdot s\(^n\)
\[ MI \] melt index, g/10 min
\[ n \] power-law parameter
\[ p \] pressure, Pa
\[ S \] screw lead, m
\[ T \] temperature, °C
\[ t \] time, s
\[ T \] temperature, K
\[ v \] velocity, m s\(^{-1}\)
\[ X \] solid bed width, m
\[ Y \] rectangular coordinate (channel depth direction), m
\[ Z \] = 1/S (turn), m\(^{-1}\)
\[ \dot{\gamma} \] shear rate, s\(^{-1}\)
\[ \eta \] apparent viscosity, Pa \cdot s
\[ \theta \] feed angle, °
\[ \rho \] density, kg m\(^{-3}\)
\[ \tau \] shear stress, Pa

Indices

\[ a \] solid
\[ b \] barrel or bulk
\[ d \] dissipated
\[ f \] flight
\[ m \] melt
\[ s \] screw
\[ t \] total
\[ x \] width channel direction
\[ z \] length channel direction

1. Introduction

The plasticating extruder is one of the basic devices used in the manufacture of plastics and man-made fibres. The growth in the production of these materials and the need to make better use of raw materials and energy require the development of better designs for extruders. The best way to fulfill this task is to develop a mathematical model of the screw extrusion process that reveals the relationships between the geometry of the screw, operating conditions and performance with the aim of obtaining the maximum throughout of the melt at a prescribed temperature, pressure and viscosity.

2. The mathematical model

A mathematical treatment of extrusion can be based on various methods. For example, Marshall et al. [1–3] used regression analysis to fit a system of empirical equations to data obtained from an experimental extruder. These equations, however, can only be used to describe the performance of other machines of similar size working with similar materials. This treatment also adds very little to the fundamental understanding of the extrusion process.

More universal are the methods based on principles of continuum mechanics. In this case, however, the extruder must be divided into several zones, to make mathematical description of the extrusion process
possible. In formulating a mathematical model, it is best to subdivide the extruder in such a way that each zone contains a characteristic physical process.

For the formulation of the mathematical model the extruder was divided into the following zones [4]:

The zone under the hopper,
the solids conveying zone,
the delay zone,
the melting zone,
the melt conveying zone.

The mathematical description of a particular zone and its exact solution depend on a number of simplifying assumptions. Except in relatively trivial cases, the resulting equations can only be solved numerically using a digital computer. Therefore, in the choice of suitable mathematical models for the various zones a compromise must also be made between cost and accuracy. The various zones are discussed below and examples of calculated profiles included for the extruders described in Appendix A.

2.1 The zone under the hopper (ZUH)

This is the part of the extruder under the hopper. The length of the zone is equal to the inside diameter of the hopper neck. The barrel is usually cooled here. It can be assumed that the solid polymer in this zone is heated by hot air from the extruder to an average of the temperatures of the barrel, screw and solids in the hopper. The pressure is given by the solid polymer hydrostatic pressure [5].

2.2 The solids conveying zone (SCZ)

In this zone the solids movement is given by different values of the friction coefficient for the solids-barrel and solids-screw regions. The mathematical model has been developed with the aid of Tadmor and Broyer's theory [6] for two-dimensional, non-isothermal, non-isotropic plug flow of solid material. The ratio of normal stresses for calculating the non-isotropic pressure distribution coefficients [7] has been taken to be 0.4 according to Schneider's theory [8]. Examples of temperature and pressure profiles are shown in figures 1 and 2.

Friction coefficient-temperature equations are given by Chung's [9] and Dekker's [10] experiments, see figure 3. In the calculation it has been assumed that the friction coefficient on the screw-root is only 70% of the value calculated for the given temperature due to a rougher surface on the barrel than on the screw. The relations between bulk density and pressure were obtained from experiments described in [12], see figure 4.

Fig. 1. Calculated solid bed temperature profiles in the SCZ for extruder no. 3 (see Appendix A)

Fig. 2. Calculated axial pressure distribution in the SCZ for extruder no. 3

Fig. 3. Calculated friction coefficient profiles in the SCZ for extruders no. 1 (solid line) and no. 3 (dashed line)

Fig. 4. Calculated axial solid plug bulk density profiles in the SCZ for extruders no. 1 (solid line) and no. 3 (dashed line)