AERODYNAMIC CHARACTERISTICS OF V-SHAPED WINGS WITH SHOCK WAVES DETACHED FROM LEADING EDGES AT HYPERSONIC SPEEDS

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The direct problem of hypersonic flow past a V-shaped wing with a shock wave detached from the leading edges is solved. The reduced normal force coefficient and the lift-drag (L/D) ratio are calculated for a configuration with a lower part in the shape of a V-wing and a streamwise upper part.

1. Consider a symmetrical flow past a V-wing with aperture angle $\gamma$ and apex angle $\beta$ (Fig. 1)

\[ b = \tan \beta \cos \gamma, \quad h = \tan \beta \sin \gamma, \quad \gamma_1 = \frac{1}{2} (\pi - \gamma) \] (1.1)

Let the function $y = y(x, z)$ represent the shape of the shock wave. Using the conservation laws, we can obtain the following expressions for the velocity components $u$, $v$, $w$ along the $x$, $y$ and $z$ axes, the pressure $p$ and the density $\rho$ behind the shock front divided by the free-stream velocity, double ram pressure and density, respectively:

\[ u' = \cos \alpha - f' y_x', \quad v' = -\sin \alpha + f', \quad w' = -f' y_z' \] (1.2)

\[ f' = (1 - \varepsilon) \frac{\sin \alpha + \cos \alpha y_x'^2}{1 + y_x'^2 + y_z'^2}, \quad p' = f' \left( \sin \alpha + \cos \alpha y_x' \right) + \frac{1}{\chi M^2}, \quad \rho' = \frac{1}{\varepsilon} \]

Here, $\chi$ is the specific heat ratio and $M$ is the free-stream Mach number. The small parameter $\varepsilon$, which is that of thin shock layer theory and equal to the ratio of the densities before and behind the shock wave, is defined by the following relation:

\[ \varepsilon = \frac{\chi - 1}{\chi + 1} \left[ 1 + \frac{2}{(\chi - 1) M^2 \sin^2 \alpha} \right] \]

We shall assume that $(\chi - 1)M^2 \sin^2 \alpha \geq O(1)$ and $\cos \alpha = O(1)$. Then, in accordance with (1.2), $u' = O(1)$, $v' = O(\varepsilon)$, $y_x' = O(\varepsilon \tan \alpha)$. We also have an estimate for the scale length of the conical vorticity flow in the compressed layer for uniform flow behind the plane shock attached to the leading edge: $z/x = O(\varepsilon \tan \alpha)$. As follows from this estimate, the transition from the flow regime with a shock attached to the wing leading edges to that with a detached shock takes place when (see Fig. 1)

\[ b = O (\sqrt{\varepsilon} \tan \alpha) \] (1.3)

Moreover, the thin shock layer theory determines the limits within which $h$ may vary (Fig. 1)

\[ h \leq O (\varepsilon \tan \alpha) \] (1.4)

Then, in accordance with (1.1), (1.3), and (1.4), we can obtain the following estimate: $\gamma_1 \leq O(\varepsilon)$. Let us assume that the shock is attached to the leading edges, this corresponding to the relation $y_x' = h - by_x'$. Then, using the impermeability condition on the wing surface $v^2/w^2 = h/b$ and taking (1.2) into account, we get

\[ y_x'^2 + \left( \frac{b \cot \alpha - \frac{h}{b}}{b} \right) y_x' + \varepsilon - h \cot \alpha = 0 \] (1.5)

where only the main terms of the expansion have been kept.

It follows from (1.5) that flow with a detached shock wave will occur when the following inequality holds:


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The quantities $C$ and $\Omega$ in (1.6) are similarity parameters determining the perturbed flow over low-aspect-ratio wings (1.3). The first of these parameters was derived in [2] for a flat delta wing, while the second was introduced in [3] for $\gamma_1 < 0$ (1.1). If the inequality (1.6) is not satisfied, then for a plane shock attached to the leading edge we have:

$$
-2 < \Omega + C < 2, \quad \Omega = \frac{b}{\sqrt{\varepsilon} \tan \alpha}, \quad C = \frac{h}{\sqrt{\varepsilon} b}
$$

(1.6)

In (1.7) the plus sign corresponds to a weak shock at the edge, and the minus sign to a strong one. We have $\gamma_2^* = 0$ when $\Omega C = 1$ (cf. (1.5)). In this case we have the design flow conditions, with a shock wave in the plane of the V-wing's leading edges [4, 5].

Figure 2 presents some domains in the plane of the parameters $C$, $\Omega$ which correspond to various conditions of flow past a V-wing. The range of the functions $C$ and $\Omega$ on which, according to (1.6), flows with a shock wave detached from the leading edges may occur, is bounded by the straight lines 1 and 2. The hyperbola $BAD$ corresponds to the design flow conditions, its branch $AB$ relating to a weak family of shocks attached to the leading edges and the branch $AD$ to a strong one. Flow regimes with a shock detached from the leading edges occur on the range of functions $C$ and $\Omega$ bounded by the straight line 2 and the curve $AD$ [6].

The diagrammatic representation of the various flow regimes over V-wings in the $C$, $\Omega$-plane (Fig. 2, $C > 0$, $\Omega > 0$) is the hypersonic analogy of the mapping of flow regimes over wings of given geometry in the plane of the parameters $M$ and $\alpha$ [7]. The solution of the direct problem of hypersonic flow past a flat low-aspect-ratio delta wing with a shock wave detached from the leading edges (Fig. 2, $C=0$, $0<\Omega<2$) was constructed and calculated for small $\Omega$ in [2]. An algorithm of the numerical solution of the system of functional equations [2] was given in [8] for all $\Omega$, $0<\Omega<2$. The theory of [2] was extended to the case of a wing with aperture angle $\gamma > \pi$ ($C < 0$, diamond-shaped wings) in [3] and to that of a wing with $\gamma < \pi$ in [9]. However, the method used in [3] and [9] to calculate these flows is valid only for very small $\Omega$.

Later, the theory of [2] was used to construct a solution of the problem of hypersonic flow past a wing with a shock wave attached to the leading edges [10–12]. However, though a solution in closed form can be obtained for the inverse problem of flow past a wing [10], when solving the direct problem of flow past a triangular plate [11, 12] unremovable singularities arise in the outer solution (using the terminology of [13]).