Dispersion Theory of Nucleon Transfer Reactions
Induced by Heavy Ions

Sh.S. Kajumov, A.M. Mukhamedzhanov, and R. Yarmukhamedov
Institute of Nuclear Physics, Uzbek Academy of Sciences, Tashkent, USSR

Received May 16, 1988

A new approach, the distorted wave pole approximation (DWPA) with the three-body Coulomb effects, is developed by combining the dispersion method and DWBA to analyse the heavy ion-induced neutron transfer reactions. The influence of the three-body Coulomb dynamics on the peripheral partial wave amplitudes is investigated. Differential cross sections of the neutron transfer reactions are calculated to compare the proposed model with the conventional DWBA. The values of nuclear vertex constants for virtual separation of neutron from various nuclei are obtained. The results of the calculations show that DWPA can be applied to analyse the heavy ion-induced neutron transfer reactions and that the three-body Coulomb effects are taken into account with acceptable accuracy in DWBA.

PACS: 24.10.Eq; 25.70.-z; 25.70.Cd

1. Introduction

Quasielastic nucleon transfer reactions are known to serve as one of the main tools to define the spectroscopic information. They can be analysed applying two different theoretical approaches:
1. the conventional DWBA;
2. the dispersion approach, e.g. the peripheral model (PM) [1].

Both approaches are in fact the various realizations of the same idea, i.e., quasielastic transfer reactions are governed by the nearest to the physical region \((-1 \leq z \leq 1\)) singularity of the reaction amplitude in \(z = \cos \Theta\)-plane (\(\Theta\) is the scattering angle in the c.m.s.). The dominant role played by the nearest singularity is the result of the surface nature of these reactions. Both methods have their advantages and disadvantages. The dispersion approach allows one to take accurately into account the contribution of the nearest singularity and the three-body effects, in particular, the three-body Coulomb effects. At the same time, in the DWBA the distortion effects in the initial and final states are accounted more accurately than it is the case in the PM. The expression for the PM differential cross section is simpler than that for the DWBA one, especially if one considers the recoil effects (RE) and the finite-range effects (FRE).

For these reasons, it is of interest to derive the expression for the reaction amplitude within the so-called hybrid model. Here the contribution from the nearest singularity is taken into consideration quite accurately as it is done in the dispersion PM, but the distortion effects in the initial and final states are kept in mind as it is done in the DWBA. Besides, the influence of the three-body Coulomb effects on the peripheral partial wave amplitudes giving the dominant contribution to the heavy ion-induced reactions amplitude has been also explicitly considered. This allows one to treat the important issue in all approaches, i.e., the extent to which three-particle Coulomb effects influence the spectroscopic information deduced from the analysis of heavy ion-induced transfer reactions. The hybrid model developed here may be called the pole approximation with the distortion waves in the initial and final states (DWPA). The relationship between the dispersion approach and DWBA is discussed in Sect. 2. Sections 3 and 4 contain the derivation of the expression for the
2. Dispersion Approach and DWBA

In this paper we consider the neutron transfer reactions

\[ X + A \rightarrow Y + B \]  

within the framework of the three-body model, i.e., we assume that \( X = A + n \) and \( B = A + n \), where \( Y \) and \( A \) are structureless particles. In the dispersion approach the reaction (1) mechanism is given by one or the sum of few simplest Feynman diagrams whose singularities in the \( z \)-plane are the nearest to the physical region \([2]\). The neutron transfer reaction mechanism is given by the sum of the pole (Fig. 1a) and triangular (Fig. 1b) graphs. The expression for the amplitude of the sum of these diagrams is

\[ \mathcal{M}_0(k_f, k_i) = M_p(k_f, k_i) + M_t(k_f, k_i), \]  

\[ M_p(k_f, k_i) = \langle k_f, \varphi_{A\text{n}} | V_{A\text{n}} G_0 V_{Y\text{n}} | \varphi_{Y\text{n}}, k_i \rangle, \]  

\[ M_t(k_f, k_i) = \langle k_f, \varphi_{A\text{n}} | V_{A\text{n}} G_0 t_{Y\text{A}} G_0 V_{Y\text{n}} | \varphi_{Y\text{n}}, k_i \rangle. \]

Here \( M_p(M_t) \) is the on-shell amplitude of the pole (triangular) diagram, \( \varphi_i(k_i) \) is the relative momentum of particles \( X \) and \( A \) (\( Y \) and \( B \)), \( \langle \varphi_{i\text{n}} | \varphi_{j\text{n}} \rangle \) is the initial (final) plane wave, \( \varphi_{ij} \) is the bound state wave function of particles \( i \) and \( j \), \( V_{ij} = V_{ij}^p + V_{ij}^A \) is the sum of the nuclear and Coulomb interaction potentials of particles \( i \) and \( j \);

\[ t_{Y\text{A}} = V_{Y\text{A}} + V_{Y\text{A}} G_{Y\text{A}} V_{Y\text{A}}, \]  

\[ G_{Y\text{A}} = G_0 + G_0 t_{Y\text{A}} G_0 = (E - T - V_{Y\text{A}} + i \varepsilon)^{-1}, \]  

\[ G_0 = (E - T + i \varepsilon)^{-1}, \quad \varepsilon \rightarrow +0, \]

is the operator of the free three-particle (\( Y, A \) and \( n \)) Green’s function, \( T \) is the kinetic energy operator of the three-particle system,

\[ E = E_i - \varepsilon_{Y\text{n}} = E_f - \varepsilon_{A\text{n}} \]

is the total energy of the three-particle system, \( E_i = k_f^2/2\mu_{X\text{A}} \) and \( E_f = k_f^2/2\mu_{Y\text{A}} \) are the relative kinetic energies of particles \( X \) and \( A \) (in the initial state) and \( Y \) and \( B \) (in the final state), respectively, \( \mu_{ij} = m_i m_j/(m_i + m_j) \), \( m_i \) is the mass of the particle \( i \), \( \varepsilon_{Y\text{n}} = m_Y - m_n - m_X \) and \( \varepsilon_{A\text{n}} = m_A + m_n - m_Y \) are the binding energies of \( X \) and \( B \). The pole amplitude \( M_p \) has the pole singularity at \( z = \zeta \), where

\[ \zeta = \frac{(k_f^2 + k_i^2 + \kappa_x^2)/2}{k_f} \]  

\[ = \frac{(k_f^2 + k_i^2 + \kappa_x^2)}{2k_i} k_f. \]

Here

\[ k_{i1} = (m_Y/m_X) k_i, \quad k_{f1} = (m_A/m_B) k_f, \]  

\[ \kappa_x^2 = 2\mu_X \varepsilon_{Y\text{n}}, \quad \kappa_B^2 = 2\mu_B \varepsilon_{A\text{n}}. \]

The singularity (9) is the nearest one to the physical region. The triangular graph amplitude \( M_t \) has also the pole singularity just at \( z = \zeta \) \([3]\). The part of \( M_t \) containing the Coulomb t-matrix \( t_{Y\text{A}} \) has this singularity. One can easily test this statement replacing \( t_{Y\text{A}} \) by \( V_{Y\text{A}}^2 \) thus obtaining the closed expression for \( M_t \). According to \([3]\), the amplitude \( M_t \) may be written as

\[ M_t(k_f, k_i) = [-1 + D(k_f, k_i)] M_p(k_f, k_i) + \tilde{M}_t(k_f, k_i), \]

where at \( z = \zeta \) \( \tilde{M}_t \) is the nonsingular part of \( M_t \). Its singularities are located farther from the physical region than \( \zeta \). The factor \( D(k_f, k_i) \) is given by the expression \([4]\)

\[ D(k_f, k_i) = \left[ (m_X m_A \varepsilon_{Y\text{n}}^{1/2} + (m_Y m_X \varepsilon_{A\text{n}}^{1/2} + (1)(m_Y m_X E_{Y\text{A}})^{1/2})^{1/2} \right] \]  

\[ E_{Y\text{A}} = [(m_Y + m_A) E + m_B \varepsilon_{A\text{n}} + m_X \varepsilon_{Y\text{n}}]/m_{Y\text{A}}, \]

\[ m_{Y\text{A}} = m_Y + m_A, \quad \eta_{Y\text{A}} = Z_Y Z_A e^{2\mu_A/(2\mu_A E_{Y\text{A}}^{1/2})}. \]

Then the amplitude \( \mathcal{M}_0 \) takes the form

\[ \mathcal{M}_0(k_f, k_i) = D(k_f, k_i) M_p(k_f, k_i) + \tilde{M}_t(k_f, k_i). \]

It is seen from (11) that summing the pole and triangular graphs of Fig. 1, one can get the renormalized pole diagram amplitude \( M_p \). This is the result of the Coulomb interaction between particles \( Y \) and \( A \) during neutron transfer. The first term in the right hand side of (11) defines correctly the pole term of the reaction mechanism taking into account three-body Coulomb dynamics. The reaction mechanism is the reaction amplitude without the distortion effects in the initial and final states. All the diagrams more complicated than the ones in Fig. 1 have the singularities...