Flux diffusion in high-\(T_c\) superconductors

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In type-II superconductors in the flux flow \((J_\perp \gg J_0)\), flux creep \((J_\perp \approx J_0)\), and thermally activated flux flow (TAFF) \((J_\perp \ll J_0)\) regimes the induction \(B(r, t)\), averaged over several penetration depths \(\lambda\), in general follows from a nonlinear equation of motion into which enter the nonlinear resistivities \(\rho_\perp(J_\perp, J_\parallel, T)\) caused by flux motion and \(\rho_\parallel(B, J_\parallel, T)\) caused by other dissipative processes. \(J_\perp\) and \(J_\parallel\) are the current densities perpendicular and parallel to \(B, B = |B|,\) and \(T\) is the temperature. For flux flow and TAFF in isotropic superconductors with weak relative spatial variation of \(B\), this equation reduces to the diffusion equation

\[ \rho_\perp \nabla^2 B + \text{correction term} \]

which vanishes when \(J_\parallel = 0\) (this means \(B \times \nabla \times B = 0\)) or when \(\rho_\perp = 0\) (isotropic normal conductor). When this diffusion equation holds the material anisotropy may be accounted for by a tensorial \(\rho_\perp\). The response of a superconductor to an applied current or to a change of the applied magnetic field is considered for various geometries. Such perturbations affect only a surface layer of thickness \(\lambda\) where a shielding current flows which pulls at the flux lines; the resulting deformation of the vortex lattice diffuses into the interior until a new equilibrium or a new stationary state is reached. The a.c. response, in particular the frequency with maximum damping, depends thus on the geometry and size of the superconductor.

1. Introduction

In high-\(T_c\) superconductors the flux lines may jump over pinning barriers by thermal activation in a rather large range of temperatures \(T\) near the superconducting transition temperature \(T_c\). This observation is due to the small coherence length, which leads to a small activation energy of pinned vortex cores, and to the large \(T\) values of these oxides. The spontaneous depinning processes lead to giant flux creep \([1–2]\), with a logarithmic time dependence of the magnetization after a sudden change of the applied field \(H_a\), and to thermally assisted flux flow (TAFF) \([3]\), with a linear current-voltage characteristic but with a strongly temperature dependent resistivity \(\rho_{TAFF}\) which is much smaller than the usual flux-flow resistivity \(\rho_{FF}\). Flux flow occurs at large current densities \(J \gg J_c, J = |J|, J_c\) is the critical current density), thermally assisted flux flow is observed at \(J \ll J_c\), and flux creep at \(J \approx J_c\), namely when the vortex lattice is in a critical state where the gradient of the induction \(B = |B|\) attains a critical value \(|\nabla B| \approx \mu_0 J_c\) (in SI units, \(\mu_0\) is the susceptibility of the vacuum). A phenomenological theory describing these three phenomena in terms of a nonlinear resistivity \(\rho_\perp(B, J, T)\) is given in Sect. 2.

The aim of this paper is to derive an equation of motion for the induction \(B(r, t)\) of the vortex state which determines the a.c. susceptibility of the superconductor in a rather general case (Sect. 2). This partial differential equation turns out to be highly nonlinear even in the cases of flux flow and TAFF where the only material parameter which enters, \(\rho_\perp\), is independent of \(J\). When the spatially varying part of \(B\) is much smaller than the average value of \(B\) this equation of motion may be linearized and reduces to the diffusion equation derived by Kes et al. \([3]\), with an additional term which vanishes if the current density has no component parallel to \(B\).

Various situations where this flux diffusion determines the a.c. response of the superconductor are discussed in Sect. 4. In general, when the applied field \(H_a\) is changed by \(\delta H_a(t)\) or when a current \(I(t)\) is applied, where \(\delta H_a(t)\) or \(I(t)\) are step functions \(\propto \delta(t)\), then the vortex lattice initially (at \(t = 0\)) interacts with this perturbation only at the surface of the specimen. This is so because in the bulk any additional current can flow only when the vortex lattice is distorted, but this distortion takes some time due to the viscous motion of the vortices. In all geometries the interaction may be described either by a change of the boundary conditions (the tangential field component must be continuous) or by a surface current which shields the interior of the superconductor from the field change or from the applied current.

A type-II superconductor with a vortex lattice behaves initially (and even at all times when the vortices...
are rigidly pinned) as a type-I superconductor in the Meissner state. The only difference is that for a type-II superconductor the penetration depth \(\lambda\) of the magnetic field is replaced by a slightly larger effective penetration depth \(\lambda' = \lambda/(1-b)^{1/2}\) where \(b = B/\mu_0 H_c2\), with \(H_c2\) the upper critical field \([4]\). When \(b\) is small one may write \(\lambda\) instead of \(\lambda'\).

The surface current \(I_s\) initially flows in a layer of thickness \(\lambda'\) (its density decreases exponentially into the interior) and exerts a force \(\phi_0 I_s \times \hat{n}\) on the vortex ends where a quantum of flux \(\phi_0\) leaves the surface with normal vector \(\hat{n}\). This Lorentz force causes the vortices to move viscously relative to the atomic lattice, with a viscosity \(\eta = B^2/\rho_\perp\) per volume. During this motion the induced \(B(r,t)\) changes such that the current density penetrates into the interior, or, in the picture of the boundary condition, the abrupt jump of the local equilibrium field \(H = B/r\) is smoothed \((B = B/B)\). In many cases this Lorentz force-driven motion of flux obeys a diffusion equation with diffusivity \(D = \rho_\perp/\mu_0\) (Sect. 3).

The diffusive nature of flux flow and TAFF in high-\(T_c\) superconductors was pointed out recently by Kes et al. \([3]\) and by Esquinazi when he tried to reconcile the “depinning lines” obtained at different frequencies by standard a.c. susceptibility measurements and by vibrating reeds \([5-7]\). Two consequences of flux diffusion are that the response of the vortex system to a step-like perturbation decreases with a characteristic time \(\tau_0 = L^2/\pi^2 D\), where \(L\) is a characteristic length, width, or thickness of the superconductor, and that for periodic perturbations \(\propto \exp(i\omega t)\) the dissipation has a maximum near \(\omega = 1/\tau_0 = \pi^2 D/L^2\). This diffusion implies that the a.c. susceptibility \(\mu(\omega)\) of a type-II superconductor in an applied field in general does not obey a Debye law \(\mu \propto 1/(1+i\omega\tau)\). The particular form of \(\mu(\omega)\) depends on the geometry and size of the system: Each eigenmode of \(B(r, t)\) [e.g., each Fourier component \(\exp(ikx)\)] decays exponentially with a different rate \(1/\tau(k) = k^2 D\) and contributes a term \(\propto \exp(-t/\tau(k))\) to \(\mu(\omega)\); \(\tau_0\) is the largest of the decay times. The sum of all these terms exhibits a more complicated time or frequency dependence. For example, a slab in a parallel field exhibits a magnetization change which may be approximated by \(\propto \exp[-(t/\tau_0)^{0.8}]\) \([3]\), and for high frequencies \(\omega \gg 1/\tau_0\) it exhibits \(\mu(\omega) \propto (1+i) \omega^{2-1/2}\) similar to the skin effect in conductors: Since at large \(\omega\) the a.c. signal penetrates only the skin depth \(\delta = (2D/\omega)^{1/2}\), the a.c. losses decrease more slowly than for a bulk effect.

### 2. Nonlinear conductivity

The resistivity of a superconductor containing vortices in general is nonlinear, and it is anisotropic even in isotropic materials due to the presence of vortices. The motion of vortices caused by a Lorentz force density \(\mathbf{J} \times \mathbf{B}\) depends only on the component \(\mathbf{J}_\perp\) of the current density \(\perp\) to \(\mathbf{B}\). The electric field generated by vortex motion, \(\mathbf{E}_\perp = \mathbf{B} \times \mathbf{v}\) is also perpendicular to \(\mathbf{B}\). For general anisotropic materials the perpendicular (or flux drift) resistivity \(\rho_\perp\) defined by \(\mathbf{E}_\perp = \rho_\perp \mathbf{J}_\perp\) is a tensor. Similarly, a parallel resistivity \(\rho_\parallel\) is defined by \(\mathbf{E}_\parallel = \rho_\parallel \mathbf{J}_\parallel\) where \(\mathbf{E}_\perp = \mathbf{B}(\mathbf{E}_\parallel \mathbf{B})\) and \(\mathbf{J}_\parallel = \mathbf{B}(\mathbf{J}_\parallel \mathbf{B})\) are the components of \(\mathbf{E}\) and \(\mathbf{J}\) parallel to \(\mathbf{B}\). The Hall effect in the superconductor shall be disregarded here; the Hall angle is of the same order as in the normal metal and thus very small, see the review papers on vortex motion \([8]\).

For usual flux motion the longitudinal field \(\mathbf{E}_\parallel\) is zero. A finite \(\mathbf{E}_\parallel\) may, however, be caused by any dissipative process other than flux-line motion, e.g., by mutual cutting of vortices followed by partial annihilation (shortening) and reconnection of vortex segments \([9]\). Vortex cutting may occur when \(J_\parallel = |\mathbf{J}_\parallel|\) exceeds a longitudinal critical current density \(J_c\) at which the vortex lattice becomes unstable to the spontaneous formation of helical instabilities \([10-13]\) with subsequent dissipative (periodic or chaotic) nucleation and annihilation of vortex helices. Though much more complicated microscopically, this threshold behavior is macroscopically similar to the usual flux flow, which occurs when \(J_\parallel = |\mathbf{J}_\parallel|\) exceeds the perpendicular critical current density \(J_p\).

Both resistivities \(\rho_\parallel\) (\(B, J_\parallel, T\)) and \(\rho_\perp\) (\(B, J_\parallel, T\)) are thus nonlinear, i.e., they depend on the current density. A tensor description of \(\rho_\parallel\) or \(\rho_\perp\), however, applies only to linear resistivities, e.g., to \(\rho_\parallel\) in the cases of flux flow or TAFF. In these cases the flux motion in anisotropic superconductors like the high-\(T_c\) oxides, for arbitrary orientation of \(\mathbf{B}\) with respect to the crystal lattice, is adequately described by treating \(\rho_\parallel\) as a tensor. In the following I shall disregard material anisotropy and consider only scalar resistivities \(\rho_\parallel\) and \(\rho_\perp\).

When thermal activation may be neglected one has \(\rho_\perp = 0\) for \(J_\parallel \leq J_{00}\), and for \(J_\parallel > J_{00}\) approximately

\[
\rho_\perp \approx \rho_{FF} \sqrt{1 - J_{cs0}^2 / J_0^2},
\]

where \(J_{00}\) is the critical current density at \(T = 0\) and \(\rho_{FF}\) is the flux flow resistivity,

\[
\rho_{FF}(B, T) = g(b) \rho_n.
\]

In (2) \(\rho_n\) is the normal resistivity at the same temperature and at \(b = 1\) where \(b = B/\mu_0 H_c2(T)\). The function \(g(b) = 0.33b\) for \(b < 1\) \([14]\) and \(g(b) = 1 - 5.2(1-b)\) for \(1 - b < 1\) \([15]\) is obtained from the time dependent Ginzburg-Landau theory (see also \([16]\)). For thermally activated flux motion one may write \([1-2, 17-23]\)

\[
\rho_\perp \approx \rho_{T\perp}(B, J_\parallel, T) = (2 \rho_n J_\parallel / J_0) \exp(-U/k_B T) \cdot \sinh(J_\parallel / J_0 k_B T),
\]

where the resistivity \(\rho_n\) at \(J_\parallel = J_0\), the activation energy \(U(B, T)\), and the critical current density \(J_0(B, T)\) may be interpreted as phenomenological parameters defined by (3). Microscopic interpretations of these parameters depend on the considered pinning model and introduce more parameters whose physical meaning is not completely clear; interpretations were suggested for various pinning models \([1-3, 21-23]\).

Equation (3) is based on the assumption of thermally activated jumps of flux lines (or of correlated volumes or “bundles” of flux lines) over pins with an effective