It is shown that in the realization of functions of logical algebra by means of circuits in automaton bases, the asymptotic behavior of the Shannon function can depend on many parameters of an arbitrarily large number of basis elements.

The problem of determining the asymptotic behavior of the Shannon function in the realization of the functions of logical algebra by means of formulas and circuits in an arbitrary basis of functional elements has been completely solved by O. B. Lupanov [1, 2], who proved that in this case the asymptotic behavior of the Shannon function depends on a parameter (the reduced weight) of one basis element.

In the case of the realization of bounded-determinate (BD) operators by means of circuits in an arbitrary automaton basis, the problem of determining the asymptotic behavior of the Shannon function becomes rather difficult, and it has been solved only in some special cases. B. A. Trakhtenbrot [5] solved it for bases consisting of functional elements and unit delay elements. He also showed [6] that the asymptotic behavior of the Shannon function depends on the reduced weights of two basis elements. To Suan Zung [7] discovered that an arbitrarily large number of basis elements can influence the asymptotic behavior of the Shannon function. Moreover, for any natural number \( r \) he constructed a basis of \( r \) elements with equal reduced weights such that the removal of only one element would increase the lower bound of the Shannon function.

In the present work we will prove that in the realization of functions of logical algebra (BD operators) by means of circuits in an automaton basis, the asymptotics of the Shannon function can depend on many parameters (weight, number of inputs, number of outputs, the system of realizable functions, the transition table, the initial state) of an arbitrarily large number of basis elements, including single-input ones.

To each element \( B_i \) of a basis \( L \) we assign a positive number \( L(B_i) \), called the weight of this element, and to each circuit \( S \) in \( L \) we assign a number \( L_S(S) \) equal to the sum of the weights of the elements of the circuit, called the complexity of the circuit. Let \( \Omega \) be a finite set of BD operators. \( L_\Omega(\Omega) \) will denote the smallest number \( L \) such that any operator in \( \Omega \) can be realized by a circuit in \( L \) whose complexity is not greater than \( L \).** The function \( L_\Omega(\Omega) \) is called the Shannon function.

Suppose that the circuit \( S \) in an automaton basis has \( n \) inputs (corresponding to the variables \( x_1, x_2, \ldots, x_n \)) and one output. (Recall that circuits in automaton bases work at discrete moments in time \( 0, 1, 2, \ldots \)). By \( x_i(t) \) we denote the value of the \( i \)-th input to \( S \) at moment \( t \). We will say that the circuit \( S \) realizes the function \( f(x_1, x_2, \ldots, x_n) \) if for any natural number \( t \) the value of the output of the circuit at the moment \( t \) is equal to \( f(x_1(t), x_2(t), \ldots, x_n(t)) \).††

*In this article, by basis we will understand a finite system of (finite) automata.
† For the definitions of bounded-determinate operators and finite automata, see [3], for example.
‡ The algorithmic unsolvability of this problem is proved in [4].
** We assume that any operator in \( \Omega \) can be realized in \( L \).
†† Thus, the input to the circuit realizing the function does not depend on the value of the inputs at the preceding moment.
Let $\Omega_n$ be the set of all functions in the logical algebra in $n$ variables, and let $r$ be an arbitrary fixed natural number. Depending on $r$ (greater than unity), we will construct a basis $\mathfrak{B}_r$ such that the time asymptotics of the function $L_{\mathfrak{B}_r}(D_n)$ depend on many parameters of $r$ elements.

We will describe some automata as a system of transformations of the form $xq' \rightarrow zq''$, where $x$ is the input set, $z$ is the output set, $q'$ is the state of the automaton, and $q''$ is its state at the next moment. $N$ will denote the set of all input sets of the automaton. If $x \in N$ and $xq' \rightarrow zq''$ for any $x$ in $\mathfrak{B}$, then the system of transformations $\{xq' \rightarrow zq''\} (x \in \mathfrak{B})$ will be written as $(\mathfrak{B})q' \rightarrow zq''$. The system of transformations $(\mathfrak{B})q' \rightarrow zq''$ will be written $\mathfrak{B}_rq'$.