A study is made of the uniqueness class of the solution of the problem concerning reconstruction of an entire function $F(z)$ of exponential type from a knowledge of the values of its derivatives $F^{(n)}(\pm hn)$, $n = 0, 1, \ldots$ ($h > 0$).

The problem of Abel concerning reconstruction of an entire function $F(z)$ of exponential type from a knowledge of the values of its successive derivatives,

$$F^{(n)}(hn) = A_n, \quad n = 0, 1, 2, \ldots$$

is well known (see [1]). In Eq. (1) we may with no loss in generality assume that $h > 0$.

Before describing the results relating to Abel's interpolational problem (1) and the problems taken up in this note, we give a number of definitions needed for the subsequent development. It is known that for every entire function of exponential type $F(z) = \sum_{n=0}^{\infty} \frac{a_n}{n!} z^n$ the power series $\sum_{n=0}^{\infty} \frac{a_n}{n!} e^{\tau t}$ always converges in some neighborhood of the point at infinity $|t| > \sigma$, $0 \leq \sigma < +\infty$, and there defines a function $\gamma(t) = \sum_{n=0}^{\infty} \frac{a_n}{n!} e^{\tau t}$, said to be associated with $F(z)$ in the Borel sense. The functions $F(z)$ and $\gamma(t)$ are connected through the relation

$$F(z) = \frac{1}{2\pi i} \int_{\Gamma} e^{\tau t} \gamma(t) dt,$$

called the Borel transformation for $F(z)$. In Eq. (2) the integration contour $\Gamma$ is taken to be a closed Jordan curve, containing in "its interior" all the singularities of the function $\gamma(t)$. The curve $\Gamma$ divides the complex plane $C$ into interior and exterior domains, which we denote symbolically by $\text{int} \, \Gamma$ and $\text{ext} \, \Gamma$, respectively. It is clear that $\text{int} \, \Gamma \cup \text{ext} \, \Gamma = C$ and $\text{int} \, \Gamma \cap \text{ext} \, \Gamma = \emptyset$. Referring to Eq. (2) we may say that the choice of the integration contour $\Gamma$ in the Borel representation is stipulated by the sole requirement: $\text{supp} \, \gamma \subset \text{int} \, \Gamma$. Transformation (2) allows us to make the following classification of entire functions of exponential type.

**Definition.** Let $E$ be an arbitrary set, $E \subset C$. We say that an entire function $F(z)$ of exponential type belongs to the class $[1; E]$, if the set of all the singularities $S_\gamma$ of the function $\gamma(t)$, associated in the Borel sense with $F(z)$, belongs to $E$. (Briefly, $F(z) \in [1; E] \Leftrightarrow S_\gamma \subset E$.)

The presence of unity in the symbol $[1; E]$ refers to the fact that the order of the function $F(z) \in [1; E]$ is always $\leq 1$.

In case $E$ is an open domain $D$, we denote the corresponding class by the symbol $[1; D]$, and in case $E = \overline{D}$ is a closed domain, then $[1; E'] = [1; \overline{D}]$. The class of entire functions whose growth does not exceed that of the first order and which is of type $\sigma$, is here denoted by $[1; |z| \leq \sigma]$.

However, following established tradition, we shall maintain the previous notation $[1; \sigma]$ for this class.

We define the class $[1; \sigma]$ through the relation $[1; \sigma] = \bigcup_{\sigma < \rho} [1; \rho]$. It is completely obvious that $[1, \sigma) = [1; |z| < \sigma]$.
We turn now to problem (1). V. L. Goncharov showed that problem (1) has a unique solution in the class \([1; 1/h]\), and that the Abel series \( \sum_{n=0}^{\infty} F^{(n)}(hn) \frac{z-hn}{hn}^{n} \) converges uniformly to \( F(z) \in [1; 1/h] \) on any compactum \( K \subset \mathbb{C} \), thereby reconstructing \( F(z) \) from the numbers \( A_n \) of Eq. (1) (see [2], [3]). It was shown next that the Abel problem (1) has a unique solution in the class \([1; 1/h]\), where the constant \( 1/h \), which defines the type of the uniqueness class, is best possible. For the latter we offer the example of the function \( F(z) = ze^{-z/h} \in [1; 1/h] \), which is not identically zero and satisfies the equations
\[
F^{(n)}(hn) = 0, \quad n = 0, 1, 2, \ldots
\]
Moreover, it was discovered that the nonextendable uniqueness class for problem (1) is the class \([1; U]\), where the domain \( U = U(h) \) is determined in the following way:
\[
U = \{ z = \rho e^{i\phi} : 0 \leq \rho < \frac{\pi - |\phi|}{h \sin |\phi|}, \ |\phi| \leq \pi \}
\]
(in connection with this, see, for example, [3], [4]). We remark that the curve \( \partial U \), being the boundary of the convex domain \( U \), is symmetric with respect to the real axis, intersects the real axis at the point \( x = -1/h \) and the imaginary axis at the points \( \pm \pi/2h \), and has two horizontal asymptotes \( y = \pm \pi/h \) for \( x \to +\infty \). The disk \( |z| < 1/h \), of all disks of the form \( |z| < r \) contained in the domain \( U \), is maximal; its radius also determines the type \( 1/h \) of uniqueness class for problem (1) in terms of \([1; \sigma]\). Further, various authors have considered the problem of reconstruction of \( F(z) \in [1; U] \) from a knowledge of the derivatives (1). Abel's problem may also be solved in other uniqueness classes than \([1; U]\); (On the methods used to solve the problem of reconstructing \( F(z) \) from the given data (1) see, for example, [4-7].)

We turn our attention to the fact that the domain \( U \) is the maximum domain of univalence of the function \( w = ze^{-h} \), playing an important role in the solution of Abel's problem. This property of the domain \( U \) will be used in studying the following problem, which we now formulate.

Our goal in this paper is an investigation of the uniqueness class of the following interpolation problem:
\[
F^{(n)}(\pm hn) = A_{\pm n}, \quad n = 0, 1, 2, \ldots
\]
relating directly to Abel's problem (1).

The data (3) contains somewhat more information concerning the behavior of \( F(z) \) than the data (1). In this regard, the question arises as to how much more extensive the uniqueness class of problem (3) is in comparison with the class \([1; U]\), the widest uniqueness class of the Abel problem (1). This question is not a trivial one inasmuch as problems exist which, in their formulation, are close to the problems (1) and (3) and where increasing the amount of information concerning \( F(z) \) does not lead to a widening of the uniqueness class. As an example of this we cite the following well known fact: the uniqueness classes of the interpolational problems \( F(n) = A_n \) and \( F(\pm n) = A_{\pm n}, \quad n = 0, 1, 2, \ldots \), coincide.

Our main result is included in the following statement:

**THEOREM 1.** The uniqueness class of the interpolational problem (3) is \([1; U \cup -U]\), where \(-U = \{ z : -z \in U \}\).

Taking into account the symmetry of the domain \( U \) relative to the real axis, we can say that \(-U \) is the mirror image of the domain \( U \) with respect to the imaginary axis.

In terms of order and type, Theorem 1 represents a definitive result and may be formulated in the following way.

**THEOREM 2.** The class \([1; \pi/2h]\) is the uniqueness class of the interpolational problem (3). The class \([1; \pi/2h]\) does not possess this property.

In other words, in the formulation of Theorem 2 the constant \( \sigma = \pi/2h \), defining the type of the uniqueness class \([1; \sigma]\) of the problem (3), is best possible. As an illustrative example we cite the function \( F(z) = \sin \frac{\pi z}{2h} \in [1; \pi/2h] \), satisfying the Eqs. (3') with \( \{ A_{\pm n} \} \equiv 0 \).