SHORT COMMUNICATION

Use of the Mitscherlich Equation in Designing Factorial Fertilizer Field Experiments to Reduce the Number of Treatments

A swampy area of 3000 hectares in the Upper Galilee of Israel was drained recently, and this newly formed land had to be investigated with respect to requirements for N, P, and K. A study involving N, P, and K, conducted on a complete factorial basis including 6 levels of each element replicated 5 times, will require no less than $6^3 \times 5 = 1080$ plots. The dependence of yield on the levels of different fertilizers may be described by different equations. The well-known Mitscherlich Equation assumes that the relationship between log yield and a certain function of fertilizers levels is linear.

It will be shown here that in order to test whether the Mitscherlich relationship holds in this new area, a restricted number of treatment combinations may prove a sufficient “reference” for testing any of the remaining treatment combinations.

The Mitscherlich Equation is as follows:

$$\log (A - y) = \log A - c (b + x)$$

(1)

where $y =$ yield obtained with a fertilizer application of $x$.

$x =$ variable amounts of the element in the added fertilizer.

$A =$ limit value of yield $y$, when $x$ approaches infinity (maximum possible yield by addition of the particular element).

$b =$ soil content of the particular element prior to fertilizer application.

$c =$ coefficient, constant for every element (slope coefficient).

Equation (1) after transformation takes the following form:

$$y = A (1 - 10^{-c(b+x)})$$

(2)

Equation (2) can be given a general form for several elements in the fertilizer:

$$y_{NPK} = A_{NPK} [1 - 10^{-c_N(b_N+N_p)}] [1 - 10^{-c_P(b_p+P_p)}] [1 - 10^{-c_K(b_k+K_k)}]$$

(3)

where $y_{NPK} =$ yield obtained with level $n$ of N, level $p$ of P, and level $k$ of K.

$A_{NPK} =$ value of $A$ for the three elements (maximum possible yield when the levels of $k$, $p$ and $n$ approach infinity)

$c_N, b_P, b_K =$ coefficients of the specified elements

$b_N, b_P, b_K =$ amounts of N, P and K in soil before fertilization.
\[ N_n = \text{amount of element N applied at level n (parallel to the variable x at the specific level of the element } x_{N_n}) \]
\[ P_p = \text{amount of the element P applied at fertilizer level p (} x_{P_p}) \]
\[ K_h = \text{amount of the element K applied at fertilizer level h (} x_{K_h}) \]

In the event that the number of levels studied for each element is 6, we can rewrite Equation (3) in a more general form by using the following signs:

\( i = \text{levels of N from 1 to 6, where n represents one of the levels} \)
\( j = \text{levels of P from 1 to 6, where p represents one of the levels} \)
\( h = \text{levels of K from 1 to 6, where k represents one of the levels} \)

Furthermore, the following abbreviated forms can be used:

\[ \alpha_i = 1 - 10^{-c_N(b_N+N_i)} \]
\[ \beta_j = 1 - 10^{-c_P(b_P+P_j)} \]
\[ \gamma_h = 1 - 10^{-c_K(b_K+K_h)} \]

Thus, an abbreviated form of Equation (3) is obtained:

\[ y_{ijh} = A_{NPK} \alpha_i \beta_j \gamma_h \]

for every specific level of N, P, and K.

The use of Equation (4) makes possible a reduction in the number of treatments required. For convenience, Equation (4) can be expressed logarithmically:

\[ \log y_{ijh} = \log A_{NPK} + \log \alpha_i + \log \beta_j + \log \gamma_h \]

Thus, for a given situation where \( i = 1, j = 1 \) and \( h = 1 \),

\[ \log y_{1,1,1} = \log A_{NPK} + \log \alpha_1 + \log \beta_1 + \log \gamma_1 \]

As the value \( i \) increases from 1 to 6, only the second quantity in the equation changes (log \( \alpha_1, \log \alpha_2 \ldots \)), while all the other quantities remain constant. The same is true for the values \( j \) and \( h \). Thus, if we know the yields obtained only as a result of an increase of \( i \) from \( i = 1 \) to \( i = 6 \), only as a result of an increase of \( j \) from \( j = 1 \) to \( j = 6 \), and likewise only as a result of an increase of \( h \) from \( h = 1 \) to \( h = 6 \), it is possible to express all the combinations of \( i, j, \) and \( h \) by one of the known yields.

For example, according to Equation (5),

\[ \log y_{2,4,5} = \log A_{NPK} + \log \alpha_2 + \log \beta_4 + \log \gamma_5 \]

where \( i = 2, j = 4, \) and \( h = 5 \).

If we know the yields for the following combinations:
\[ y_{1,1,1}, y_{2,1,1}, y_{1,4,1} \text{ and } y_{1,1,5} \]

we can say that the sum of the yields
\[ \log y_{2,1,1} + \log y_{1,4,1} + \log y_{1,1,5} \]

will include all the quantities of \( \log y_{2,4,5} \) in Equation (5) with a surplus of twice the yield of \( y_{1,1,1} \) (which is also known to us). Consequently, we can write

\[ \log y_{2,4,4} = \log y_{2,1,1} + \log y_{1,4,1} + \log y_{1,1,5} - 2 \log y_{1,1,1} \]

or generally:

\[ \log y_{ijh} = \log y_{i,1,1} + \log y_{1,j,1} + \log y_{1,1,h} - 2 \log y_{1,1,1} \]