TRAPPING OF PARTICLES IN A TIME-DEPENDENT HAMILTONIAN

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Abstract. The trapping of particles in the case of two coupled oscillators, when the frequency of one approaches the constant frequency of the other exponentially in time, is investigated. It is found that we can predict the region of initial conditions for which a test particle can be trapped around a resonant periodic orbit.

1. Introduction

In many model problems of our Galaxy numerical calculations have shown the existence of orbits that remain in the neighbourhood of a stable periodic orbit (see, e.g., Ollongren, 1965). Test particles moving along these orbits can be considered as trapped, thus we call such orbits 'trapped'.

One type of trapped orbits has been found in a spiral galaxy near the particle resonance (Barbanis, 1970). On a plane rotating with the angular velocity of the spiral pattern, which is equal to the angular velocity of the galaxy at the corotation distance, there is a set of orbits librating around the potential maxima at that distance.

The properties of trapped orbits in time independent potentials are well understood today. However, the main problem remaining is, what happens if a perturbation (e.g., a spiral field) is introduced slowly. In such a case it is important to find how many stars become trapped and what is their final distribution.

The problem is especially difficult because the method of adiabatic invariants fails during the trapping process. One can construct adiabatic invariants valid in each particular resonant region (see, e.g., Contopoulos, 1975). However, all these adiabatic invariants do not apply in the most interesting case, namely when an untrapped particle gets trapped or vice versa. In order to understand these transition cases it is necessary to use numerical methods.

The present paper is a first step towards an understanding of such transition phenomena. The study of the characteristics of trapped orbits in the case of realistic models of galaxies is rather difficult and time consuming. Thus we decided to use a simple model that we would be able to explore in detail.

We study the case of a very simple potential field

\[ V = \frac{1}{2}(Ax^2 + By^2) - \varepsilon xy^2. \]  

(1)

This may be considered to represent the potential field on the plane of symmetry of a galaxy that has been deformed by the close passage of another galaxy. Of course such
a potential should have many more terms in order to be realistic. However, it may be considered as representing a special deformation of the central parts of an elliptical galaxy.

The corresponding Hamiltonian is

$$H = \frac{1}{2}(\dot{x}^2 + \dot{y}^2 + A x^2 + B y^2) - \varepsilon x y^2. \quad (2)$$

Several investigators have used this or similar Hamiltonians in order to explore the properties of stellar orbits (see, e.g., Hénon and Heiles, 1964; Contopoulos and Moutsouls, 1965, 1966; Hori, 1967; Contopoulos, 1968).

The cases $A = B$ and $A \approx B$ in particular represent one of the most important resonance cases. These cases can be also considered as a coupling of two oscillators with equal, or nearly equal frequency. The coupling constant $\varepsilon$ gives the deviation of this non-integrable system from the integrable one where the two oscillators are uncoupled.

In the case of the Hamiltonian (2) Contopoulos and Moutsouls (1965) found numerically five periodic orbits, three stable and two unstable. Two of the stable periodic orbits are straight lines through the origin and the third one is an oval curve.

![Fig. 1. The pattern of invariant curves (solid lines) in the surface of section $(\bar{x}, \bar{\dot{x}}; y=0)$ in the case of two coupled oscillators with equal frequencies when $A = B = 0.1$, $\varepsilon = 0.05$, $H = 0.00765$. The invariant points a, b, c, d correspond to stable periodic orbits while e to an unstable one. The outermost circle (dashed) is the invariant curve of the $\bar{x}$-axis which is an unstable periodic orbit too. The straight lines (dashed) that divide the $\bar{x}, \bar{\dot{x}}$ plane into equal sectors of $15^\circ$ are used in Figure 3.](image)