The Moment of Inertia at the Saddle Point of Low Energy Fission of Even-Even Nuclei

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We use the molecular model of low energy fission, which describes the nucleus by two interacting fragments, to calculate the moment of inertia for U$^{236}$ in the cranking approximation including BCS theory. We show that the moment of inertia at the saddle point:

1. (1) depends almost linearly on the fragment distance,
2. (2) is influenced only very weakly by the pairing constant and by the fragment deformations,
3. (3) shows, as a function of the distribution of mass between the two fragments ($A_1, A_2$), a minimum near the magic configuration $A_1 = 132, Z_1 = 50$ and depends in this mass region strongly on the term structure near the Fermi energy,
4. (4) is approximately that of a rigid body.

1. Introduction

For understanding low energy fission of nuclei in the framework of an adiabatic model, the transition state of the compound nucleus at the saddle point is of great importance. Therefore, many efforts have been made in studying this stage of the fission process. Beside the deformations at the saddle point, the moment of inertia is frequently calculated to characterize this stage, because it should be one of those quantities which can be extracted most directly out of the angular distribution of the fragments and the transition state energy spectrum, once measured. There have been several attempts to calculate the moment of inertia in the framework of the liquid drop model, also with some refinements concerning shell effects. But these calculations yields only the rigid body moment of inertia. Though we are dealing with a heavy nucleus in a very deformed state where this approximation should be reasonably good,

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we know that for even-even nuclei the values of the moment of inertia is reduced severely by taking into account the superfluidity of the nucleus. In this way, much better agreement with experimental data was obtained.

In this paper we are treating the moment of inertia on a semi-microscopic level by using the cranking approximation. For calculating the wave functions, we use a special adiabatic model, the molecular model of nuclear fission, which was introduced by Nörenberg. Since the saddle point configuration of the nucleus is not yet known in this model, we are studying the moment of inertia as a function of various parameters. As in ref. 7 the calculations are restricted to the case of even-even nuclei.

The compound nucleus (mass \(A\), charge \(Z\)) is described by two interacting fragments \((A_1, Z_1)\) and \((A_2, Z_2)\) separated by the fragment distance \(d\). For the sake of simplicity a static approximation of the molecular model is used, i.e. the ground state of the system is calculated for a given fragment distance \(d\) and fixed fragment masses \(A_1\) and \(A_2\) disregarding all collective motion. The single particle states \(|\alpha\rangle\) are expanded in terms of eigenfunctions of two unperturbed spherical fragments \(|p\rangle\):

\[
|\alpha\rangle = \sum_p c_p |p\rangle.
\]

The fragment eigenfunctions \(|p\rangle\) are localized around the centres of fragments \(F=1\) and \(F=2\). The energy expectation value with a BCS wave function is minimized, obtaining in a well known manner a coupled set of Hartree-Fock-Bogoliubov (HFB) and BCS equations. The following constraints are imposed on the variations: (1) the single particle states should be orthonormalized and (2) the number of protons and neutrons, the mass difference between the fragments, and their distance should be held constant. In order to simplify the solution of the HFB equations, the selfconsistent potential is approximated by two oscillator potentials with quadrupole and octupole deformations, localized at the fragment distance \(d\). The Coulomb potential of a uniform charge distribution with the same deformations is taken into account explicitly. In the BCS equation a constant matrix element \(G_F\) for each fragment \(F\) is introduced, yielding the following equations to determine the gap parameter \(\Delta_a\) for the state \(a\):

\[
\Delta_a = w_a^1 A_1 + w_a^2 A_2
\]

\[
\Delta_F = \frac{G_F}{2} \sum_{\beta>0} \frac{w_\beta^F A_\beta}{\sqrt{e_\beta^2 + A_\beta^2}}
\]

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