SU6 covariant S-Matrix formalism

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Starting from the assumption that the SU2 content of SU6 refers to total angular momentum instead of intrinsic spin a strictly Lorentz- and SU6 covariant S-matrix is constructed. The underlying symmetry is closely connected to the inhomogenous SL (6, c) proposed by several authors. The approach leads to a unique description how angular momentum has to be treated. It turns out that it is not sufficient to describe a p-wave by a “35” kineton except for very small particle momenta.

Introduction

The SU6-theory of Gürsey and Radicati and Sakita was spectacularly successful in its application to particles at rest. The generalization of this symmetry to finite momenta is ambiguous and so far no general accepted version exists.

The largest group of papers deals with the direct generalization of Wigner’s supermultiplet theory: spin independence is postulated where spin refers to intrinsic spins of quarks or elementary particles in a relativistically modified way. In those theories the particle momenta are parameters which are not affected by the SU6 transformations of spin and unitary spin. If, however, the SU2 subgroup contained in SU6 rotates only the spin a relativistic kinetic energy term is not invariant under the group and breaches the symmetry; the symmetry is “intrinsically broken”. Consequently, such theories can only be understood in terms of a perturbation treatment; the interaction Lagrangian may fulfil the symmetry but nothing ensures the symmetry for the

4 An illuminating discussion is given by H. J. Lipkin in a paper intitled: Now we are SU6, Rehovoth preprint.
S-matrix elements. Connected with this fact is the incompatibility of the symmetry with unitarity. It seems to us that the best way out of this dilemma is to give up the notion of an intrinsic spin independence which anyhow leads to serious difficulties in any simple quark model.

In this paper we postulate instead that the spin content of SU6 refers not to intrinsic spin, but to total angular momentum! Such a hypothesis immediately implies that orbital angular momentum and momentum have general SU6 properties. This results because the SU6 transformations will mix angular momentum with unitary spin. Therefore, unitary spin properties of space cannot be avoided. In order to carry through this idea it is necessary to embed the physical states into a larger Hilbert space. On first sight this seems to be a serious disadvantage for any such theory. In fact it is not: it only implies that for physical states, where the particles are eigenstates of spin, charge, and hypercharge operators a certain direction in the 36 dimensional space is fixed and of special significance. The same situation is familiar in isospin space, where the 3-direction is fixed by the charges of the particles.

A theory where the momentum is indeed transformed by SU6 transformations has recently been developed by Rühl and by Fulton and Wess. In these papers the SU6 group has been generalized to the full inhomogenous SL (6, c) group in a consequent way.

The approach of the present paper is closely related to this work but differs from it in some respect. We want to incorporate SU6 invariance in the S-matrix theory in the simplest possible way based on physical intuition even at the cost of mathematical beauty. In order to avoid interpretation difficulties as much as one can the Lorentz covariance of the S-matrix elements will be required for physical states only. SU6 invariance is considered in each Lorentz system simply as a generalization of rotational invariance and SU3 invariance. Through these SU6 rotations of states with physical 3-component momentum vectors unphysical momentum vectors appear which, however, have a fixed norm and lead to no obvious difficulties. In spite of these restrictions to SU6 rotations we are able to construct the S-matrix in terms of invariant amplitudes. We are aware of the fact that the restrictions imposed prevent the application of useful grouptheoretical theorems referring to the inhomogenous SL (6, c). These restrictions are