Lattice Vibrational Waves in Cubic Crystal Plates

By


With 4 Figures

(Received March 23, 1965)

Summary. An investigation is given of the lowest symmetric (extensional) and the lowest antisymmetric (flexural) mode of waves in a cubic crystal plate bounded by two planes of symmetry. The investigation was carried out using the continuum theory of anisotropic elasticity, and also using a simple cubic lattice model. Particular attention was given to the change of the character of both the extensional and flexural mode, which takes place as the wave length is decreased from infinity down to values of the order of the plate thickness. Both modes tend to become surface modes whenever surface waves are possible along the direction of propagation. However, some essential differences were observed, regarding this transition from bulk modes to surface modes, between materials which propagate Rayleigh surface waves and those which propagate generalized Rayleigh surface waves.


I. Introduction

Lattice vibrational spectra have been studied largely under the assumption of periodic boundary conditions which simplify the analysis but mask the effect of surfaces. The latter, however, may be quite important in the study of semiconductor properties some of which are strongly influenced by surface characteristics, e.g., the mobilities of electrons and holes in surface layers. Another area where the existence of surfaces should be taken into account is the study of physical properties of very small crystal particles, for example, specific heats and optical absorption coefficients of very finely powdered crystals.
The effect of surfaces on lattice vibrations of cubic crystals has been considered in two previous papers [1], [2]. In [1], a discussion was given of surface waves propagating along a (001) surface of a semi-infinite simple cubic lattice medium. In [2], an investigation was made of extensional waves in a plate of a similar medium bounded by two (001) surfaces. It was shown that as the wavelength decreases with respect to the thickness of the plate the nature of the plate waves changes from that of bulk modes to that of surface modes. This is true as long as surface modes exist along the direction of propagation, which is always the case for the (100) direction considered in [2].

In the present paper we complete and extend the results of [2] as follows: First, we treat both extensional and flexural waves. And second, we consider waves along a direction other than (100). Of particular interest is the study of materials such as Cu or Ge, for which according to [1] no surface modes exist along directions in the neighborhood of 110. It is shown that for such materials, along these directions, the displacements are never confined near the boundary, no matter how small the wavelength is taken in comparison with the plate thickness. This is true for both extensional and flexural waves.

As in [1] and [2] the investigation has been carried out first using the continuum equations of the linear theory of elasticity and then using the lattice model given in [1]. When the displacement variation as a function of the lattice position is very slow in all three directions, the lattice solution matches that of the continuum theory.

II. Continuum Theory

A. Frequency Equation

Consider an elastic plate of a cubic anisotropic continuum bounded by two parallel planes of symmetry, \( z = \pm b \). The \( x \) and \( y \) axes are taken along the principal directions. The equations of motion are those of the classical linear anisotropic elasticity given, for example, by Stoneley [3].

Let the displacement components \( u, v, \) and \( w \), along the \( x, y, \) and \( z \) directions, be

\[
(u, v) = (U, V) f_1(\kappa q z) \exp i \kappa (l x + m y - \omega t), \quad w = W f_2(\kappa q z) \exp i \kappa (l x + m y - \omega t),
\]

where \( \omega \) is the phase velocity, \( \kappa \) the wave number, \( l \) and \( m \) the direction cosines of propagation, and \( q \) a dimensionless parameter. The functions \( f_1 \) and \( f_2 \) are defined as follows: In the case of symmetric waves

\[
f_1(\tau) = \cos \tau, \quad f_2(\tau) = \sin \tau,
\]

and in the case of antisymmetric waves

\[
f_1(\tau) = \sin \tau, \quad f_2(\tau) = \cos \tau.
\]