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The Friendly Giant

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Table of Contents

1. Introduction

In this paper, we demonstrate the existence of the *Friendly Giant*, a finite simple group of order

$$2^{46}3^{20}5^9 7^6 11^2 13^3 . 17 . 19 . 23 . 29 . 31 . 41 . 47 . 59 . 71$$
$$=808{,}017{,}424{,}794{,}512{,}875{,}886{,}459{,}904{,}961{,}710{,}757{,}005{,}754{,}368{,}000{,}000{,}000.$$

Evidence for the existence of this group was produced independently in November, 1973, by Bernd Fischer in Bielefeld and by this author in Ann

Dedicated to the memory of Richard D. Brauer, February 10, 1901-April 17, 1977

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Arbor. Serious work on this group – mainly a study of subgroups and conjugacy classes – began the first weekend of that month in both locations. Additional details of this early work are discussed in Sect. 15. For now, we add only that such a simple group appeared likely to have a complex irreducible character of degree 196883; in 1974, this number was established as a lower bound for the degree of a nonprincipal irreducible character [13, 37]. While this evidence for the existence was very persuasive, it did not constitute a proof. Our existence proof was announced on January 14, 1980 and more formally in [29].

Our method is to take a 196884-dimensional module B for a particular group C of shape $(2^{1+24})(\cdot 1)$, define on B the structure of a commutative nonassociative algebra with a symmetric nondegenerate associative bilinear form, then define an automorphism σ of this algebra. The group $G=\langle C, \sigma\rangle$ is the simple group of the title (the usual symbol for this group is F_1). The extra rigidity required by expecting our linear group to preserve an algebra structure enables us to make precise definitions of the relevant linear transformations and verify their required properties. The reason we thought of this approach is the following. Simon Norton had computed the values of a hypothetical character χ of degree 196883 and computed that $(S^2\chi, 1)=1$, $(S^3\chi, 1)=1$, $(S^3\chi, \chi)=1$ and χ is rational-valued. It follows that if M is a module affording χ, M has the structure of a commutative (but not necessarily associative) algebra with a nondegenerate associative symmetric bilinear form. This finding of Norton was the inspiration for this paper. See Sect. 15 for additional comments on algebras associated to finite simple groups.

We comment on some over-all aspects of the construction. In some sense, the algebra B is described using only basic linear algebra. The group theory used is descriptive in nature. Thus, one could say that the construction of $G=\langle C, \sigma\rangle$ is elementary. That is, starting from scratch, one may construct M_{24}, then $\cdot 0$ and finally G, with each stage depending on the previous one. See two paragraphs ahead and look at Table 1.1. However, the identification of G as a finite simple group with the right properties requires deep results from the classification of finite groups. It is possible that this dependence can be eliminated, for instance, by counting configuration of vectors in B permuted by G. An enumeration of any such configurations may be long and difficult, however.

Section 2 contains various preliminary results, mainly about group representations, the Leech lattice, Conway groups and the classification of finite simple groups. Sections 3 and 4 set up basic notation. In Sect. 5, we compute the C-invariant algebra structures on the module B, and in Sect. 6 we select the one we work with in the rest of the paper (modulo a choice of F made in Sect 7). Sections 7, 8 and 9 discuss various technicalities needed both in the definition of σ (Sect. 10) and in the proof of the "main result," Proposition 11.2, that σ is an algebra automorphism. Section 7 is concerned with a choice of complement F which will cause the function β to behave well, while Sects. 8 and 9 develop techniques for analyzing the action of certain elements of C on basis elements, mainly for the purpose of being able to analyze β. Nearly all of Sect. 11 is concerned with a proof of the main result, which in turn amounts to verifying a list of identities involving configurations of vectors in the Leech