LAWS OF VIBRATIONAL COMPACTION OF DIFFICULTLY DEFORMABLE POWDER MATERIALS. II. THEORETICAL ANALYSIS OF THE VIBRATIONAL COMPACTION OF NON-PLASTIC POWDERS


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The laws of densification of difficultly deformable powders in the process of vibrational compaction were studied. It was found that the efficiency of the process depends upon the magnitude of the applied force necessary to densify the powder. The necessary force is attained by rapid deceleration of the powder mass against the upper punch. Theoretical results were in good agreement with experimental data obtained on the unit ÉRGUK.

The vibrational compaction of difficultly deformable powders was investigated using the equipment ÉRGUK (Fig. 1) developed in the Institute of Impulsive Processes and Technology of the Ukrainian Academy of Sciences. Its unique feature is use of the impulsive action of an electrical discharge as a source. A pressure impulse is transmitted to the powder from the discharge chamber through an elastic plate on which a die is attached. The upper punch is fixed by additional inertial loading. Compaction occurs by the action of the elastic element on the column of powder, situated between the bottom of the die and the lower face of the punch. Provision is made for control of the electrical discharge frequency, the interaction energy, and the magnitude of the inertial load.

We consider the laws of vibrational compaction of a powder ingot in a type ÉRGUK unit on the basis of the model of a free-flowing medium described in [1], and compare the theoretical results with the experimental data.

According to [1], the velocity of motion of the powder is determined by minimizing the function

\[ W = \int_{\Omega} \varphi(\dot{u}) d\Omega - G(\ddot{u}). \]  

(1)

Here

\[ \varphi(\dot{u}) = \int_0^1 D_\Sigma (\sigma_\alpha, \alpha_\gamma) \frac{d\alpha}{\alpha}; \quad D_\Sigma = Q + D_p + D_{pv} + D_v \]

with respect to \( \ddot{u}_p, \ddot{u}_v, \) where \( \ddot{u}_p \) is the velocity of motion of the powder due to repacking of its particles, and \( \ddot{u}_v \) is the velocity due to approach of the particles without change in the packing.

The actual velocity of the particles is the sum of the two components:

\[ \dot{u} = \ddot{u}_p + \ddot{u}_v. \]  

(2)

Compaction of the powder occurs under the action of the lower punch, moving at the velocity

\[ u = u_0(t), \]  

(3)

where \( u_0(t) \) is an experimentally determined function of the deflection of the plate (elastic element) in the process of vibrational compaction. A force equal to the weight of the inertial load is applied to the upper punch. The calculations are based on the assumption that the velocity of radial motion is negligible compared to the longitudinal velocity, i.e., we consider one-dimensional motion of the loosely poured medium in space.
As noted in [1], and follows from equation (2), the total deformation velocity of the medium is the sum of two terms \((e_{ij}^\text{P} \text{ and } e_{ij}^\text{v})\) corresponding to different mechanisms of deformation. In a one-dimensional system we have one nonzero component in the deformation velocity tensors for each of these, which we designate as \(e_p\) and \(e_v\). We assume that the total deformation velocity of the material is

\[ e_{ij} = e_{ij}^\text{P} + e_{ij}^\text{v}. \]  

We attempt to determine the ratio of \(e_{ij}^\text{v}\) and \(e_{ij}^\text{P}\) at which the function (1) is a minimum. It will be a minimum if the expression under the integral sign in (1) is a minimum at every point in the volume. Considering the additional condition (4), and using LaGrange multipliers \(\lambda_{ij}\), the extremal conditions take the form

\[ \frac{\partial \phi}{\partial e_{ij}^\text{P}} + \lambda_{ij} = 0, \quad \frac{\partial \phi}{\partial e_{ij}^\text{v}} + \lambda_{ij} = 0. \]  

The function \(\phi\) is the potential for determining stress as a function of the deformation velocity [2]; therefore, considering (5), we obtain \(\sigma_{ij}^\text{P} = \sigma_{ij}^\text{v}\). Thus, if the deformation velocity field satisfying equation (5) is kinematically possible, then the stresses connected with repacking and approach of the particles are mutually equal. In a one-dimensional system this statement is valid, and a stress \(\sigma\) exists in the material such \(\sigma^\text{P} = \sigma^\text{v} = \sigma\). The rheological diagram for deformation of a granular material is given in Fig. 2. It represents viscous and visco-plastic elements connected in parallel. In the general case, the condition of equality of the components of the stress tensors \(\sigma_{ij}^\text{P}\) and \(\sigma_{ij}^\text{v}\) is not satisfied. However, in the isotropic media under consideration

\[ p^\text{P} = p^\text{v} = p, \quad \tau^\text{P} = \tau^\text{v} = \tau, \]

where \(p\) and \(\tau\) are the invariants of the stress tensor. Deformation of a granular medium proceeds as a result of approach of the particles until the stress reaches a limiting value at which repacking can begin. After repacking the average distance between particles increases, and then they again begin to approach, and etc.

Numerical minimization of equation (1), and determination of the velocity of motion in SiC powder, was carried out by the finite element method. The value of \(D_{ij}\) [1] was obtained from the experimentally determined function \(p(\sigma)\) for the uniaxial compression of powder with an average particle size of 50 \(\mu\)m (Fig. 5). The kinetic temperature \(T\) in the volume of the ingot was considered constant, and found from the equilibrium condition \(\xi = 0\), where \(\xi = Q - D_p - D_v - D\). As follows from the assumptions made in the basic model, the total energy input in compaction can not be less than that for