Phase Transition of First Order in Superconductors at $H_{c1}$

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The Ginzburg-Landau-Functional is extended by nonlinear terms. The corresponding Ginzburg-Landau equations are solved by a method analogous to Abrikosov's theory near $H_{c2}$. But here it yields the magnetization as a function of $H_{ex}$ in the whole vortex phase presuming that $\kappa$ is near $1/\sqrt{2}$. At $H_{c1}$ a jump in the magnetization is found.

1. Introduction

Recent experiments$^{1-6}$ on type II superconductors with $\kappa$ near $1/\sqrt{2}$ have shown a phase transition of first order at $H_{c1}$. Changing the external field $H_{ex}$, the magnetic induction $B$ shows a jump $B_0$ depending on temperature and $\kappa$. $B_0$ seems to tend to zero with $T \rightarrow T_c$ faster than $H_{c1}$ and $H_{c2}$. Therefore the Ginzburg-Landau equations are not expected to yield the jump$^7$. To explain it, one would extend the Ginzburg-Landau theory, for example either include the next terms $\sim T_c-T$ as Tewordt did$^8$ (see also$^9$), or extend phenomenologically the London equation$^{10}$ or rely on the full Gorkov equations from the beginning$^{11,12}$.

Some authors$^{7-11}$ have calculated the structure of an isolated flux line, taking into account higher orders of the nonlocality. But just as the vortex structure arises from the nonlinear character of the Ginzburg-Landau equations in addition to the nonlocal, the nonlinear extension of the Ginzburg-Landau functional must also be expected to influence the penetration of the flux at $H_{c1}$.

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Our aim is to verify this suggestion. Disregarding the solution for an isolated vortex line (so we cannot say anything about a possible attraction), we solve the Ginzburg-Landau equations, extended by nonlinear terms, with a method\textsuperscript{13} analogous to Abrikosov's theory\textsuperscript{14} near $H_{c_2}$. This suggests itself and becomes possible because with the measured discontinuous penetration of the flux for $\kappa$ near $1/\sqrt{2}$ the condition $|\psi|^2 \leq |\psi_0|^2$, necessary for Abrikosov's method, holds true in the whole vortex region $H_{c_1} \leq H \leq H_{c_2}$. The latter condition also restricts the range of the $\kappa$ values, for which our results are valid.

As Abrikosov's theory makes essential use of the fact that the physical free energy is minimal, we first write down a generalized Ginzburg-Landau functional (Sec. 2). In essence it is Tewordt's functional but renormalized to be stable. In Sec. 3 we solve the field equations. As the solution we find $H_{c_1}/H_{c_2}$ as function of $\kappa$ (Sec. 4). Moreover we determine the jump $B_0/H_{c_1}$. Finally, the latter is calculated in the spirit of the presented method but using earlier results of Eilenberger, based on the Gorkov equations.

2. Extension of the Free Energy

As a possible extension of the Ginzburg-Landau functional, one at first thinks of Tewordt's expansion. It is, however, unstable and thus cannot be used as a suitable starting point for minimum principles. On the other hand it should show the typical properties regarding the order of the nonlinearity. Therefore we start with the form of the functional obtained by the asymptotic expansion up to the first order in $T_c-T$, but we take into account the influence of the higher orders by a renormalization of the values of the constants.

Thus we take the following free energy:

\[
\mathcal{F} = \mathcal{F}_N + \int d^3 \vec{r} \left\{ -\tilde{\alpha} |\psi|^2 + \frac{\tilde{\beta}}{2} |\psi|^4 
+ \left( \frac{\hbar}{2} \right) \left( \frac{2e}{m^*} A \right) \psi^2 \left( \tilde{\gamma} + \tilde{\sigma} |\psi|^2 \right) + \tilde{\xi} (\frac{\hbar}{2} \frac{H^2}{8\pi}) \right\}.
\]

(1)

The quantities $\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma}$ are renormalized constants corresponding to the usual expansion coefficients. Neumann and Tewordt\textsuperscript{8} found $\tilde{\alpha} - \alpha \sim \tilde{\beta} - \beta \sim \tilde{\gamma} - \gamma \sim \tilde{\xi} - \xi \sim T_c - T$. The quantities $\tilde{\sigma}$ and $\tilde{\xi}$ are the coefficients of the new terms. We have omitted the nonlocal term of higher order in Tewordt's functional\textsuperscript{8}. Even without it, we find a transition of first order at $H_{c_1}$.

\begin{itemize}
  \item \textsuperscript{13} Saint-James, D., Thomas, E.J., Sarma, G.: Type II superconductivity. Pergamon Press 1969.
  \item \textsuperscript{14} Abrikosov, A.A.: Soviet Phys. JETP 5, 1174 (1957).
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