A Remark on the Coupling of the $p$-Trajectory to the $\pi\pi$ System

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Received January 30, 1969

A finite energy sum rule for the $\pi\pi$ scattering suggests that the $p$-trajectory chooses nonsense at the point $\alpha_p = 0$, and that the spin of the $g(1650)$ meson is $J^P = 1^-$. In this note we discuss the implications of an appropriate finite energy sum rule $^1$ (FESR) satisfied by the $\pi\pi$ scattering amplitude. We consider the $T=1$ $\pi\pi$ scattering amplitude in the $t$-channel. Analyticity in the $s$-channel combined with crossing symmetry lead then to the following FESR:

$$
\sum_{T} \alpha_{T,1} \int_{v_0}^{N} \text{Im} A^T(v, t) \, dv = \beta^{(0)}(t) \frac{N^{\alpha_p + 1}}{\alpha_p + 1}
$$

where $v = \frac{s-u}{2}$ and the crossing matrix elements $\alpha_{T,1}$ have the following numerical values, $\alpha_{0,1} = \frac{2}{3}$, $\alpha_{1,1} = 1$, $\alpha_{2,1} = -\frac{2}{3}$. In writing Eq. (1) we have assumed that in the region $s > N$ ($\sqrt{s}$ is the total energy in the $s$-channel), the scattering amplitude is dominated by the exchange of the $p$ Regge trajectory and $\beta^{(0)}(t)$ is the corresponding residue function, representing the coupling of the $p$-trajectory to the $\pi\pi$ system ($\sqrt{-t}$ is the momentum transfer in the $s$-channel). The low energy spectrum on the left of (1) is assumed to be dominated by the known resonant states which are coupled strongly to the $\pi\pi$ system ($p, f, g$) and also by a strong interaction in the $T=J=0$ channel, represented in general by a large $S$-wave phase shift, $\delta^0(s)$, in the low energy region ($4m_n^2 < s < m_p^2$). The $p$-meson contributes to the $T=1$ amplitude on the left of Eq. (1) and from the experiments we have $m_p = 760$ MeV, $\Gamma_p = 120$ MeV. The $f$-meson contributes to the $T=0$ amplitude and its parameters are $m_f = 1260$ MeV and $\Gamma_f = 140$ MeV. Finally, the $g$-meson is less well known experimentally. It has been observed in the high energy region $^2$ ($m_g \approx 1650$ MeV) and

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its total width is $\Gamma_g=170$ MeV. It has isospin $T=1$ and there is evidence that it decays strongly into two pions. The spin of the $g$-meson is not established experimentally and the possible spin-parity assignments are $J^p=1^-,3^-,...$ consistently with the Bose statistics of the two pion system with $T=1$. We observe that the $t$-dependence of the Eq. (1) may be sensitive to the value of the spin of the $g$-meson and therefore one could be able to draw some relevant conclusions using the FESR (1). We distinguish two possibilities. $J^p=1^-$ or $3^-$, excluding the higher spins since the corresponding states could not be accomodated in a reasonable Regge pole pattern in the Chew-Frautschi diagram. In the first case ($J^p=1^-$) the various contributions on the left of (1) are written as follows ($m_\pi=1$)

$$\text{Im} A_T^0(v,t) = \left(\frac{s}{s-4}\right)^{\frac{1}{2}} \sin^2 \delta_0^0(s) + 5\pi m_f \Gamma_f P_2 \left(1 + \frac{2t}{s-4}\right) \delta(s-m_f^2),$$

$$\text{Im} A_T^1(v,t) = 3\pi [m_\rho \Gamma_\rho \delta(s-m_\rho^2) + m_g \Gamma_g \delta(s-m_g^2)] P_1 \left(1 + \frac{2t}{s-4}\right)$$

where $P_j$ denotes the Legendre polynomial and $\sin^2 \delta_0^0(s)$ is different from zero and large only in the low energy region ($4m_\pi^2<s<m_\rho^2$). The contribution of the $T=2$ interaction is neglected and we have assumed that the coupling of the $g$-meson to various inelastic channels (other than $\pi\pi$) is very small. This last conjecture will be discussed in connection with our final results. If the spin of the $g$-meson is $J^p=3^-$, the structure of the term $\text{Im} A_T^1(v,t)$ in (2) is modified as follows:

$$\text{Im} A_T^1(v,t) = 3\pi m_\rho \Gamma_\rho \delta(s-m_\rho^2) P_1 \left(1 + \frac{2t}{s-4}\right)$$

$$+ 7\pi m_g \Gamma_g \delta(s-m_g^2) P_3 \left(1 + \frac{2t}{s-4}\right).$$

Using the formulae (2) and (3) we may write Eq. (1) as follows:

(a) If $J^p=1^-$

$$I_0 + \frac{10\pi}{3} m_f \Gamma_f P_2 \left(1 + \frac{2t}{m_f^2-4}\right) + 3\pi m_\rho \Gamma_\rho P_1 \left(1 + \frac{2t}{m_\rho^2-4}\right)$$

$$+ 3\pi m_\rho \Gamma_\rho P_1 \left(1 + \frac{2t}{m_\rho^2-4}\right) = \beta^{(p)}(t) N_{p+1}^{a+1}.$$

(b) If $J^p=3^-$

$$I_0 + \frac{10\pi}{3} m_f \Gamma_f P_2 \left(1 + \frac{2t}{m_f^2-4}\right) + 3\pi m_\rho \Gamma_\rho P_1 \left(1 + \frac{2t}{m_\rho^2-4}\right)$$

$$+ 7\pi m_g \Gamma_g P_3 \left(1 + \frac{2t}{m_g^2-4}\right) = \beta^{(p)}(t) N_{p+1}^{a+1}.$$