Orbital Rearrangement in the $sd$-Shell Nuclei

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The single particle orbital rearrangement energy is found to lie between 30 and 50% of the BRUECKNER rearrangement energy in the $sd$-shell. This calculation shows that the orbital rearrangement energy $R_e$ is in a good approximation equal to half the difference of the single particle energies $e_\sigma$ in the system with $A$ particles and in the system with $A-1$ particles having a hole in the state $|\sigma\rangle$.

1. Introduction

The nuclear rearrangement effect was first studied by BRUECKNER and coworkers in nuclear matter. Recently KÖHLER reinvestigated the problem. The average experimental value has been calculated from the separation energies of the protons and neutrons by ELTON. Since the theoretical investigations were all made in nuclear matter they take into account the so called "Brueckner rearrangement". This is due to the change of the Pauli projection operator in the Brueckner-Goldstone equation coming from the removal of a particle. The orbital rearrangement which is due to the change of the selfconsistent potential is not included in the effect considered by BRUECKNER. Here we want to calculate the orbital rearrangement energy and compare the effect with the Brueckner rearrangement determined in nuclear matter. BRUECKNER et al. did not mention the orbital rearrangement. But KÖHLER estimated that it can be neglected compared to the change in the separation energy coming from the Pauli principle.

KÖHLER calculated for the differences of the $1s$ single particle energies between the $A$- and the $(A-1)$-system $e^{A-1} - e^A$ the values 15 MeV in $^{16}$O and 10 MeV in $^{40}$Ca. The rearrangement energies are approxi-
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approximately half of these values: 7.5 MeV in $^{16}$O and 5 MeV in $^{40}$Ca. (KöHLER$^2$ assumes that the separation energy $S_a$ is $-\varepsilon_a^{A-1}$. The correct value is given by the difference of the binding energies of the nuclei with $A$ and $A-1$ nucleons where in the latter a hole is in the state $|\alpha\rangle$ (see Eq. (2)). We find (Table 1 and 2) that this is approximately $S_a \approx -\varepsilon_a^{A-1} + (\varepsilon_a^{A-1} - \varepsilon_a^A)/2$. One therefore has to halve the values calculated by BRUECKNER et al. and by KöHLER to find the rearrangement energies as defined by KöHLER in the last of the Ref. 2. Our calculation yields an orbital rearrangement energy of 3.4 and 3.0 MeV, respectively.

The single particle rearrangement energy $R_a$ for a state $|\alpha\rangle$ is defined as the positive difference between the selfconsistent HF-single particle energy and the separation energy. If the projectiles stay long compared with the rearrangement time in the range of the interaction with the nucleus, the residual nucleus is left in an eigenstate of the $(A-1)$-many nucleon problem:

$$H|\Phi_a(A-1)\rangle = E_a|\Phi_a(A-1)\rangle.$$ 

In this case the separation energy $S_a = -\varepsilon_a^A - R_a$ is the difference between the binding energy of the ground state in the $A$ particle system and the excited state in the $(A-1)$-system having a hole in the level $|\alpha\rangle$.

$$S_a = A^4B(g) - A^{A-1}B(\alpha),$$

$$R_a = A^{A-1}B(\alpha) - A^4B(g) - \varepsilon_a^A = -\varepsilon_a^A - S_a.$$ 

(2)

In the other extreme where the reaction time is short compared to the rearrangement time the residual nucleus is left in the state:

$$a_\sigma|\Phi(A)\rangle = \sum_\sigma c_\sigma|\Phi_\sigma(A-1)\rangle.$$ 

(3)

This is no eigenstate of the many body Hamiltonian, but it can be expanded into such states (Eq. (3)). The separation energy is $-\varepsilon_a^A$. An estimate of the energy spread can be found from the uncertainty principle: One finds for $(p, 2p)$ at 400 MeV a width of approximately 20 MeV.

Is the reaction time comparable with the rearrangement time then we expect a separation energy between $S_a$ and $-\varepsilon_a^A$ and a width between the value due to the transition probability of the hole into other states and between the value appraised above.

In the non-adiabatic case the separation energy is equal to the HF-energy $-\varepsilon_a^A$ and can easily be calculated. In the adiabatic case the rearrangement energy (2) consists of two parts as mentioned above: The Brueckner rearrangement energy and the orbital rearrangement energy.

In section 2 we outline shortly how the separation energy is calculated as the difference of the total HF-binding energies between the ground