Simulation of heat and mass transfer in a finite porous medium using orthogonal collocation method*

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Abstract. Orthogonal collocation method was applied to solve Luikov-type model describing coupled heat and mass transfer in a finite porous body. Numerical examples show, that sometimes even one-point collocation can give sufficient results from the practical point of view, and the ratio of the simulation and real process time is considerably smaller than that of provided by finite difference method. Computed values are in good agreement with experimental findings.

Simulation des Wärme- und Stofftransportes in einem endlichen porösen Medium mit der orthogonalen Kollokation-Methode


Nomenclature

\( A \) elements of the collocation matrix \( A \)

\( a \) diffusivity (\( m^2 s^{-1} \))

\( b \) relative \( t \) (\( °C \))

\( B \) elements of the collocation matrix \( B \)

\( c_0 \) specific heat capacity (\( J kg^{-1} °C^{-1} \))

\( d_i(t) \) functions in the temperature profiles (\( °C \))

\( e_i(t) \) functions in the moisture content profiles

\( L \) thickness (m)

\( N \) evaporation rate (\( kg m^{-2} s^{-1} \))

\( p \) pressure (Pa)

\( r \) evaporation heat (\( J kg^{-1} \))

\( t \) time (s)

\( T \) temperature (\( °C \))

\( u \) moisture content (\( kg~moisture/kg~dry~solid \))

\( x \) space coordinate (m)

Greek symbols

\( \alpha_c \) heat transfer coefficient (\( W m^{-2} °C^{-1} \))

\( \phi \) phase change criterion

\( \eta \) dimensionless space coordinate

\( \lambda \) conductivity, mass (\( kg m^{-1} s^{-1} \)), heat (\( W m^{-1} °C^{-1} \))

\( \sigma \) evaporation coefficient (\( s^{-1} \))

Subscripts

\( c \) convective

\( cr \) critical

\( g \) gas

\( h \) thermal

\( m \) mass

\( s \) solid

\( w \) vapor

\( t \) total

Dimensionless groups

\( B_{l_m} \) Biot – number for mass transfer, \( \sigma p_L/\lambda_m \)

\( B_{l_h} \) Biot – number for heat transfer, \( \alpha_c L/\lambda_k \)

1 Introduction

Many times mathematical models well accepted by academic society are used very rarely or not at all in practical life. This is not because they are physically irrelevant or do not represent significant improvement in describing the real process, but because their numerical realization needs considerable mathematical skill or too much computation effort. These are the main problems with the application of the Luikov-type models describing coupled heat and mass transfer in capillary porous bodies [1], too.

In the last two decades, many analytical exact and approximate as well as numerical solutions have been proposed, but almost all of them have some considerable restrictions (linearized model equations, special boundary conditions and geometries), or unfavourable characters (very long computation time, convergence problems, big storage capacity requirement) from the point of view of practical application, i.e. [2–5].

Although orthogonal collocation was also used in attacking drying problem [6, 7], the real merit of this method in practical simulation — as far as we know — has not been demonstrated, yet.

In this paper we should like to illustrate, that orthogonal collocation technique can provide astonishingly accurate and fast solutions of the drying equations.

* Dedicated to Professor S. Endrenyi
2 Collocation equations for convective drying

Let us consider a solid porous material containing moisture in its pores. On the surfaces, it has contact with the drying gas having high temperature and low humidity enough to initiate heat and mass transfer process between the material and the gas.

Neglecting diffusion term representing mass transfer caused by temperature gradient, the heat and moisture transport inside the solid material can be described by the following equations [1]:

\[ \frac{\partial T}{\partial t} = a_h \nabla^2 T + b \frac{\partial u}{\partial t}, \]  
\[ \frac{\partial u}{\partial t} = a_m \nabla^2 u. \]  

The boundary conditions on the surface are:

\[ \alpha_c (T_g - T) = 2h \frac{\partial T}{\partial n} + r (1 - e) \sigma (p_{ws} - p_{wg}), \]  
\[ \sigma (p_{wg} - p_{w}) = 2m \frac{\partial u}{\partial n}. \]

and at the symmetry point(s) of the body:

\[ \nabla T = \nabla u = 0. \]

The partial pressure of the moisture in the gas having direct contact with the solid surface can be generally expressed as function of the surface temperature and moisture content of the porous body:

\[ p_{ws} = p_{ws} (T_s, u_s). \]

In one dimensional case, let us approximate the moisture and temperature distributions by the following polynomial expressions:

\[ T(\eta) = \sum_{i=1}^{N+1} d_i \eta^{2(i-1)}, \]  
\[ u(\eta) = \sum_{i=1}^{N+1} e_i \eta^{2(i-1)} \]

where \( \eta \) is a dimensionless distance:

\[ \eta = \frac{x}{L}, 0 \leq \eta \leq 1. \]

Supposing that these profiles satisfy Eqs. (1) and (2) in \( N \) inner collocation points: \( \eta_1, \eta_2, \ldots, \eta_N \), then the collocation equations are:

\[ \frac{dT_j}{dt} = \frac{a_h}{L^2} \sum_{i=1}^{N+1} B_{ji} T_i + \frac{a_m b}{L^2} \sum_{i=1}^{N+1} B_{ji} u_i, \]  
\[ \frac{du_j}{dt} = \frac{a_m}{L^2} \sum_{i=1}^{N+1} B_{ji} u_i \]

where \( T_j \) and \( u_j \) are the temperature and moisture content in \( \eta_j \).

At the surface, in the \( N+1 \)th collocation point, one may get from the boundary conditions, Eqs. (3) and (4):

\[ \alpha_c (T_g - T_{N+1}) = \frac{\lambda_h}{L} \sum_{i=1}^{N+1} A_{N+1,i} T_i + r (1 - e) \sigma (p_{ws} (T_s, u_s) - p_{wg}), \]  
\[ \sigma (p_{wg} - p_{w}) = -\frac{\lambda_m}{L} \sum_{i=1}^{N+1} A_{N+1,i} u_i. \]

The boundary conditions at the symmetry point(s) are automatically satisfied. Now, we have \( 2N \) linear ordinary differential equations and two algebraic ones for \( 2N + 2 \) unknown. Note, that if Eq. (6) is linear, then \( T_{N+1} \) and \( u_{N+1} \) can be eliminated, and analytic solution is possible.

For prespecified \( N \), the coordinate of the collocation points, \( \eta \) and the constant coefficients of matrices \( A \) and \( B, A_{ji}, B_{ji} \) can be found in [8], in case of different geometries.

If the functions \( T_j (t) \) and \( u_j (t) \) are known, \( j = 1, 2, \ldots, N+1 \), then the coefficients, \( d_i \) and \( e_i \) can be calculated from the following linear algebraic equation systems:

\[ T_j = T(\eta_j) = \sum_{i=1}^{N+1} d_i \eta_j^{2(i-1)}, \]  
\[ u_j = u(\eta_j) = \sum_{i=1}^{N+1} e_i \eta_j^{2(i-1)}. \]

In this way the temperature and moisture content profiles are determined.

Now let us apply these equations to a classical problem of convective drying, to batch drying of a porous slab.

3 Batch drying of a slab

The porous slab is \( 2L \) thick, and the evaporation takes place on its both surfaces, at \( x = -L \) and at \( x = L \) (see Fig. 1). Using one point collocation, the collocation equations are:

\[ \frac{dT_1}{dt} = \frac{a_h}{L^2} (B_{11} T_1 + B_{12} T_2) + \frac{a_m b}{L^2} (B_{11} u_1 + B_{12} u_2), \]  
\[ \frac{du_1}{dt} = \frac{a_m}{L^2} (B_{11} u_1 + B_{12} u_2). \]

From the boundary conditions, we get:

\[ \alpha_c (T_g - T_1) = \frac{\lambda_h}{L} (A_{21} T_1 + A_{22} T_2) + r (1 - e) \sigma (p_{ws} (T_1, u_1) - p_{wg}), \]  
\[ \sigma (p_{wg} - p_{w}) = \frac{\lambda_m}{L} (A_{21} u_1 + A_{22} u_2) \]

where \( t \) is a dimensionless distance:

\[ t = \frac{x}{L}, 0 \leq \eta \leq 1. \]