Energetical Factors in Power Systems with Nonlinear Loads

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The classical definitions for the energetical determining factors in power systems are reviewed and a new model for the apparent power is suggested. Based on the volt-ampere characteristic of the nonlinear load, an equivalent circuit, containing linear elements, can be determined. In this way, load-flow and harmonic compensation studies can be simplified.

1. Introduction

The economic attractiveness of power semiconductor switching devices for frequency or magnitude conversion and control have caused an upsurge in the application of thyristors for industrial and domestic use. This trend is far from leveling off. Superior types of rectifier values are expected to give great impetus to dc transmission and related HV devices, new load leveling systems as cryogenic inductors or fuel cells in connection with converters which will replace the conventional pumped hydro storage plants. Consequently, in the next few years, the electric utilities will face an increasing power demand from devices characterized by the risk of considerable disturbances injected into the network [1].

The quality of electric energy cannot be related today to the stability of the frequency and the amplitude of the voltage only, and utilities as well as consumers are considering the degree of deviation of the wave from its ideal sinusoidal shape. Notions as Waviness [2], Maximum Theoretical Deviation from a Sine Wave [3], Distortion Factor are frequently used to express in a concise way the degree of harmonics content. Other definitions as Telephone Influence Factor, Equivalent Disturbing Voltage or Current are key parameters in the design of adequate filters and the evaluation of the effect on the adjacent communication systems.

Due to the versatility of the thyristor, new techniques of power conditioning are used and new types of perturbations are faced by the utility engineer [4]. The integral cycle control was associated with the problem of lamp flicker. The nonsymmetrical firing of thyristors is causing d.c. components and subharmonics. The actual definitions and standards do not cover all the power supply aspects of semiconductor equipment. There is need to elaborate new criteria and rules that will assure in the future the electric energy quality and a fair cooperation between the consumer and utilities. The goal of this work is to review the classical definitions used for the energetical values related to networks with nonlinear impedances, and to suggest and analyze alternative methods of approach that can lead to a better evaluation of the energetical determining factors.


When a nonsinusoidal periodic voltage

\[ v(t) = \sum_k \sqrt{2} V_k \sin (\omega_k t - \alpha_k); \quad \omega = 2 \pi/T \]  

is impressed across a nonlinear impedance, the instantaneous load current is

\[ i(t) = \sum_u \sqrt{2} I_u \sin (\omega_u t - \beta_u). \]
The harmonics spectrum will be identified using the following sets of indexes
\[ N = \{0, 1, 2, \ldots, k, \ldots, n\} \]
for voltage harmonics and
\[ M = \{0, 1, 2, \ldots, u, \ldots, m\} \]
for current harmonics.

The set \( N \) of voltage harmonics can be divided into two sub-sets \( G \) and \( P \), \( N = G \cup P \). The sub-set \( G \) represents the intersection of set \( N \) with the set \( M \) of current harmonics, \( N \cap M = G \). From here results that for the most general case the set \( M \) of current harmonics contains two sub-sets too, \( M = G \cup R \). The sub-set \( G \) of current harmonics is present within the voltage spectrum while the sub-set \( R \) of current harmonics is not included in the voltage spectrum.

Based on the above description one can divide the current in three components.

\[
i(t) = \sum_{uG} \sqrt{2} I_u \cos (\beta_u - \alpha_u) \sin (u\omega t - \alpha_u) - \\
- \sum_{uG} \sqrt{2} I_u \sin (\beta_u - \alpha_u) \cos (u\omega t - \alpha_u) + \\
+ \sum_{uR} \sqrt{2} I_u \sin (u\omega t - \beta_u) .
\]

Therefore, the instantaneous electromagnetic power can be considered as a superposition of three components related to the current’s division.

\[
p_a(t) = v(t) i(t) = \hat{p_a}(t) + \hat{p}_t(t) + \hat{p}_d(t) .
\]

Where the first term

\[
\hat{p_a}(t) = \sum_{uG} V_u I_u \cos (\beta_u - \alpha_u) \left[1 - \cos 2(u\omega t - \alpha_u)\right] \quad (3)
\]
is positive for passive loads and its average value is known as active power or effective power.

\[
P = \frac{1}{T} \int_{0}^{T} \hat{p_a}(t) \, dt = \sum_{uG} V_u I_u \cos (\beta_u - \alpha_u) . \quad (4)
\]

The second and third terms are summations of sinusoidal waves with double frequencies \(2u\omega\) as well as the combinations \((u \pm k)\omega\)

\[
\hat{p_t}(t) = - \sum_{uG} V_u I_u \sin (\beta_u - \alpha_u) \sin [2(u\omega t - \alpha_u)] 
\]
and

\[
\hat{p_d}(t) = \sum_{kG \atop uG} \sum_{uG} V_{ku} \{ \cos [(u - k) \omega t + \alpha_k - \beta_u] - \\
- \cos [(u + k) \omega t - \alpha_k - \beta_u] \} + \\
+ \sum_{kN \atop uR} \sum_{uR} V_{ku} \{ \cos [(u - k) \omega t + \alpha_k - \beta_u] - \\
- \cos [(u + k) \omega t - \alpha_k - \beta_u] \} .
\]

Those components load the lines giving no contribution to the transport of energy. In a fully compensated system, i.e., the power factor \(PF = 1\), \(\hat{p}_d(t) = 0\). For the sinusoidal case \(\hat{p}_d(t) = 0\) and \(\hat{p}_t(t) = V I \sin (\beta - \alpha) \cdot \sin [2(\omega t - \alpha)]\) and its amplitude equals the reactive power.

C. Budeanu [6, 7], based on the mathematical definitions

\[
S^2 = V_{\text{RMS}}^2 I_{\text{RMS}}^2 = \sum_{uG} V_u^2 \sum I_u^2 ,
\]
presented in 1927 the model by which the apparent power is considered to be the vector sum of the effective power and the fictitious power \(S^2 = P^2 + Q^2\). Further the fictitious power \(F\) was divided into two components in quadrature such that finally \(S^2 = P^2 + Q^2 + D^2\), where

\[
Q = \sum_{uG} V_u I_u \sin (\beta_u - \alpha_u) \quad (5)
\]

Traditionally this term was called reactive power, and

\[
D^2 = \sum_{kG \atop kN \atop uG} I_{ku}^2 + \sum_{kN \atop kR \atop uR} I_{ku}^2 + \sum_{kG \atop kR \atop uR} I_{ku}^2 + \sum_{kG \atop kG \atop uG} I_{ku}^2 \quad (6)
\]
is known as distortion power or harmonic power. Budeanu’s model was popularized and applied to the energetic studies of rectifier circuits by H. Rissik [8, 9] and today is universally accepted [2, 10].

In order to characterize polyphase systems, this model was expanded [11, 12] and for the general case of asymmetrical polyphase systems with ground wire, the apparent power has a fourth component, the asymmetry power: \(S^2 = P^2 + Q^2 + D^2 + A^2\).

For example, if a three phase transmission line with the equivalent \(r\) ohms/phase respectively \(r_o\) resistance of the neutral wire supplies a nonlinear load, the asymmetry power [11] is given by the relation.

\[
A^2 = \frac{r_o}{r_{\phi \phi}} \sum_{uG} (V^{(u)}_k)^2 \sum_{uG} (I^{(u)}_k)^2 ; \quad \phi = a, b, c \quad (7)
\]

where \(\phi\) refers to the phases \(a, b, c\) and \(I^{(u)}_k\) is the rms harmonic current of order \(u\) within the neutral wire.

Fig. 1 presents typical waveforms of voltage, current and instantaneous power for a phase controlled rectifier bridge. The instantaneous powers and their average are the representative values of the energy flow from the physical standpoint. Moreover, the fictitious powers \(Q\) and \(D\), correlated to \(\hat{p}_t\) and \(\hat{p}_d\), are mathematical definitions enabling the reference to a figure of merit for a certain consumer. When the voltage wave is sinusoidal, the quantity \(Q\) is equal to the amplitude of \(\hat{p}_t\). Fig. 1d. For non-