Energy Flow in Ultra-Relativistic Nucleus-Nucleus Collisions and Comparison between Models

T. Ochiai
Department of Physics, Rikkyo University, Tokyo, 171 Japan

Received 4 December 1986

Abstract. We calculate the cross sections of transverse and neutral energy production in the nucleus-nucleus collisions using the additive quark model (AQM) and the wounded nucleon model (WNM). In the case of $\alpha \alpha$ collisions, the experimental data of the transverse energy ($E_T$) distribution favor the WNM, whereas those of the total neutral energy and the transverse neutral energy distributions favor the AQM. We predict the $E_T$ distributions in the heavy ion collisions using these two models. The AQM and the WNM give different results.

1 Introduction

There has been increasing interest in ultra-relativistic nucleus-nucleus collisions. The heavy ion experiments at the CERN-SPS and the BNL-AGS will soon begin. In these experiments, formation of the quark gluon plasma (QGP) [1] is expected. It is believed that the QGP is produced if high energy density or high baryon number density is attained. High energy nucleus-nucleus collisions are also important in order to investigate the mechanism of hadron production.

The transverse energy ($E_T$) flow will be measured in the experiments [2]. Transverse energies of all particles produced in a nucleus-nucleus collision sum up to the $E_T$. Since central nucleus-nucleus collisions cause the large $E_T$ flow, the cross section at the large $E_T$ may be sensitive to the abnormal phenomena such as the QGP formation. So there is need to predict the $E_T$ distribution using current theoretical models. Comparison of the calculations with the experimental data will be useful to detect the abnormal phenomena and also to select the correct model. The predicted $E_T$ distribution become the background of the abnormal phenomena if they take place.

Already in the preliminary data [3] of the proton-Pb collisions at $p_{lab}=200$ GeV/c at the CERN-SPS, the $E_T$ distribution extends to 50 GeV far beyond the kinematical limit (= 19.4 GeV) of the proton-nucleon collision. And also the $E_T$ and the total neutral energy ($E^0_{TOT}$) distributions in $\alpha \alpha$ collisions at the CERN-ISR at $\sqrt{s_{NN}}=31$ GeV are observed [4, 5] beyond the nucleon-nucleon ($N-N$) kinematical limit.

In this paper, we present the calculations of the $E_T$, $E^0_{TOT}$ and the transverse neutral energy ($E^0_T$) distributions in $\alpha \alpha$ collisions using the additive quark model (AQM) [6] and the WNM [7]. Our results are that the AQM can reproduce the $E^0_{TOT}$ and $E^0_T$ distributions, while the AQM overestimates the $E_T$ distribution. We also predict the $E_T$ distributions in heavy ion collisions using these two models. These models give different results for the $E_T$ distribution.

In Sect. 2, we derive the formulae for the $E_T$ distribution. In Sect. 3, we show our results. Section 4 is devoted to the conclusion.

2 Formulation

2.1 The Energy Distribution by the Additive Quark Model

Here, we give the formulae for the energy ($E$) distribution in nucleus-nucleus collisions according to the AQM [6]. In this section, $E$ means $E_T$, $E^0_{TOT}$ or $E^0_T$. We begin with the case of the proton-proton ($p-p$) collision. A quark in the projectile proton interacts with a quark in the target proton, and then a colored string spans between them. Fragmentation of the colored string contributes to the particle production in the central rapidity region. In the case of the proton-nucleus collisions, several colored strings span as the result of several quark-quark interactions. However, colored strings attached to the same quark in the projectile proton interact and collapse into one col-
ored string. Therefore, the number of the colored strings is identical to that of the wounded quarks in the projectile proton. We assume that the number of the quarks in the proton is three \[6\]. In the nucleus-nucleus collisions, many colored strings span at the initial stage. Among them, colored strings connected through the wounded quarks interact with each other and collapse into one colored string.

Bialas and Kolawa \[8\] estimated the distribution of the number of the colored strings in $\alpha$-$\alpha$ collisions using the classical probability calculus. It is difficult, however, to apply their method to the cases of no small nuclei. We calculate the distribution of the number of the colored strings using an approximate method which can also be used for large nuclei.

In the nucleus $(A)$-nucleus $(B)$ collisions, $(A \leq B)$, the probability that $w$ quarks in the nucleus $A$ are wounded is given by \[9\]

$$P_{AB}(w) = \left(\frac{3A}{w}\right) \int d^2 b \left(\{p_A(b)\}^w \{1 - p_A(b)\}^{3A-w}/\sigma_{AB}^{\text{norm}}\right), \quad (2.1)$$

where

$$p_A(b) = \left[ \int d^2 b \left[1 - \left\{1 - \sigma_{qq} t_A(b)\right\}^{3A}\right]\right], \quad (2.2)$$

and $\sigma_{AB}^{\text{norm}}$ is the normalization, which is numerically near the inelastic cross section of the $A-B$ collisions. We take the quark-quark cross section to be $\sigma_{qq} = (1/9) \sigma_{NN}$ \[6\]. Here, $\sigma_{NN}$ is the $N-N$ inelastic cross section. The nuclear thickness $t_A(b)$ is defined by

$$t_A(b) = \int dz \rho_A(r), \quad (2.3)$$

where $\rho_A(r)$ is the nuclear density normalized to unity. We use the gaussian nuclear density for $A = 4$,

$$\rho_A(r) = (\pi R^2)^{-3/2} \exp\left(-r^2/R^2\right), \quad (2.4a)$$

where $R = 1.37 \text{ fm}$ \[7\], and the Woods-Saxon nuclear density for other nuclei,

$$\rho_A(r) = \rho_0 [1 + \exp\{t - R_A/d\}], \quad (2.4b)$$

where $R_A = 1.19 A^{1/3} - 1.61 A^{-1/3} \text{ fm}$ and $d = 0.54 \text{ fm}$ \[10\].

We assume that among $w$ wounded quarks, the number of quarks which are connected with each other through the colored strings at the early stage after the collisions is equal to the number of the wounded quarks in nucleus $A$ in the quark-nucleus $(q-A)$ collisions \[5\]. These quarks, the number of which we write $m$, are eventually connected to one colored string. In this case, the number of the colored strings in the $A-B$ collisions is $w/m$ (which is not an integer in general, see below). Therefore, the probability that $n$ colored strings span in the $A-B$ collisions is

$$b(n) = \sum_{w=1}^{3A} \sum_{m=1}^{3A} \left\{\left(1 + \left[w/m\right] - w/m\right) \delta_{n,w/m}\right\}$$

\[2.5\]

$$\cdot P_{AB}(w) Q_{qA}(m),$$

where $\left[w/m\right]$ is the Gauss' notation. Here, the probability that $m$ quarks in nucleus $A$ are wounded in the $q-A$ collisions is given by

$$Q_{qA}(m) = \left(\frac{3A}{m}\right) \int d^2 b \left[\sigma_{qq} t_A(b)^m \right]$$

\[2.6\]

$$\cdot \left\{\left[1 - \sigma_{qq} t_A(b)^{3A-m}\right]/\sigma_{qq}^{\text{norm}}\right\},$$

where

$$\sigma_{qq}^{\text{norm}} = \left[ \int d^2 b \left[1 - \left\{1 - \sigma_{qq} t_A(b)^{3A}\right\}\right]\right], \quad (2.7)$$

is the inelastic cross section of the $q-A$ collisions. The first term of the right hand side in (2.5) is introduced because $w/m$ is not an integer in general. Even in the case of $w/m < 1$, at least a quark is wounded. So, we assume that a colored string spans in this case. Therefore, the probability of the event having one colored string, $b(1)$, is at least $P_{AB}(1)$. In Fig. 1, we show the distribution of the number of the colored strings, $b(n)$, for $\alpha$-$\alpha$ collisions. The distribution is consistent with that of Bialas and Kolawa \[8\].

The $E$ distribution of the event having $n$ colored strings is given by

$$g_n(E) = \int dE_1 \ldots dE_n g_1(E_1) \ldots g_1(E_n) \cdot \delta(E - E_1 - \ldots - E_n). \quad (2.8)$$

In the AQM, the $E$ distribution by one colored string is assumed to be equal to that in the $p-p$ collisions. Experimental data of the $E$ distribution in the $p-p$