Eigenvalues of the Faddeev Equation Kernel for a System of Three Spinless Particles

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For a system of three spinless particles interacting via separable Yamaguchi potential the possibility of the existence of three-particle resonances is studied. To this end, the eigenvalues $\lambda(E_3)$ of the Faddeev equation kernel have been calculated in the c.m.-energy region $-30 \text{ MeV} \leq E_3 \leq 15 \text{ MeV}$ for total momentum states $L=0$ and $L=1$. It is shown that in the investigated energy range there are no resonances.

1. Introduction

To calculate the eigenvalues and eigenfunctions of the kernel of the Faddeev equations [1] is important at least for two reasons, firstly for the search for resonances in three-body systems and secondly for the separable expansion of the three-particle amplitude. The theoretical investigation of three-particle resonances represents a poor elaborated branch of the three-particle problem as compared e.g., with the study of the bound states. This is caused on the one hand by mathematical difficulties in solving the corresponding Faddeev equations and on the other hand by the absence of unambiguous experimental indication of the existence of such resonances in nonrelativistic systems. In brief, the experimental situation in systems with $A=3$ [2, 9] may be characterized by the fact that if resonances exist, then they are produced with very small cross sections and, probably, have large widths. Up to now it is difficult to understand the experimental results from the theoretical point of view, since it is very little known on the conditions under which three-particle resonances appear and on the sensitivity of their physical characteristics to a change of the two-particle interaction parameters. Concerning the importance of the investigation of three-particle resonances one should remember that at present such systems are the only multiparticle systems with several decay channels for which, starting from a given two-particle interaction, accurate calculations of resonance states can be performed. Moreover the question of three-particle resonances is of interest in elementary particles physics to explain the existence of mesons like $A_{1,2}$, $C$, $D$, $E$, $\omega$ on the basis of our knowledge on the interaction between the elementary particles.

As to the second point mentioned above, concerning the separable expansion of the three-particle amplitude, it is of great importance in connection with the solution of the four-body problem, since the three-particle amplitude forms the kernel of the Faddeev-Yakubovsky equations for four particles [3]. The separable expansion of these kernels allows one [4, 5] to essentially simplify the complicated four-particle equations. The separable expansion may also be used for the calculation of vertex coupling constants of three and four-particle systems which are important characteristics of bound states, together with the binding energy and the mean square radius.

The properties of resonances in a system of three elementary particles have been studied in papers [7] on the basis of a relativistic version of the Faddeev equations. The authors have found a considerable sensitivity of the eigenvalues to the shape of the two-particle interaction. Resonances in a model approach with one heavy particle ("nucleus") and two light ones $(n, p)$ for negative total energy have been investigated in detail in papers [15]. Eigenvalues and eigenfunctions of a system of modified three-particle Lippmann-Schwinger equations with Gaussian-type two-body potential have been calculated in papers [8]. The existence of resonances in a system consisting of three neutrons has been investigated in paper [10]. Two opposite cases with respect to the two-particle interaction have been considered: a large-range square well...
potential and a $\delta$-potential. Resonances were shown to be absent in both cases. A promising approach in the study of three-particle resonances permitting an extension to the relativistic case was proposed in [11]. Starting from the three-particle unitarity relations and using the $N/D$-method an integral equation has been derived for the scattering amplitude describing the scattering of a particle on a two-particle resonance. This integral equation turned out to be very suitable for the investigation of the conditions for the existence of three-particle resonances.

As is known, the mathematical difficulties in the investigation of three-particle resonances consist in the fact that at positive total energy there appear moving singularities of logarithmic type in the kernel of the Faddeev equations. Recently methods have been proposed for solving this problem [12-14]. However, up to now these methods have only been applied to scattering problems and not to resonances.

In the present paper we calculate the eigenvalues and eigenfunctions of the Faddeev-kernel for a system of three spinless particles interacting via separable Yamaguchi potential using the mathematical methods [13, 14]. In Section 2 we present the initial equations and give a definition of the three-particle resonances. In Section 3 we show for a sufficiently general class of separable potentials that it is possible to transform the kernel of the integral equation in such a way that after interpolation of the solution the remaining singular integrals can be calculated analytically. In Section 4 we present the numerical results of the calculation.

2. The Homogeneous Faddeev Equation and Three-Particle Resonances

We start from the amplitude $T(f, k; k_0, z_3)$ describing the following process

$$T(f, k; k_0, z_3) = \sum_{k'} f_{k_0}^{k'} T(k', k; k_0, z_3)$$

For three identical spinless particles the Faddeev equation [1] may be written in the form

$$T(f, k; k_0, z_3) = t_s(f, p_2, \bar{z}_2) \varphi_2(p_{10})$$

$$+ \frac{1}{(2\pi)^3} \int \frac{dk'}{k'} T(p_1, k'; k_0, z_3) \left[ z_3 - \frac{\hbar^2}{m} (k^2 + k k' + k'^2) \right]$$

(1)

where $z_3$ is the total c.m.-energy of the three-particle system (in what follows $z_3$ takes any complex value), $\bar{z}_2 = z_3 - (3/4)(\hbar^2/m)k^2$, $p_{10} = (k + k_0, 0)$, $p_{20} = (k/2 + k_0, 0)$, $p_1 = (k + k/2)$, $p_2 = (k/2, k')$, $\varphi_2$ is the wave function of the two-particle bound state ("deuteron") and $m$ — the nucleon mass. The symmetrized two-particle $t$-matrix is defined as $t_s(k', k, z_2) = t(k', k, z_2) + t(-k', k, z_2)$. The matrix elements $t(k', k, z_2)$ obey the equation

$$t(k', k, z_2) = v(k', k) + \frac{1}{(2\pi)^3} \int \frac{dk''}{z_2} \frac{v(k', k'') t(k'', k', z_2)}{2(1 - k'^2/m)}$$

(2)

where $v(k', k) = \langle k' | v | k \rangle$ with $\langle k' | k \rangle = (2\pi)^3 \delta(k' - k)$.

The quantity $v$ is the two-particle potential. Separating the angular variables and allowing for interaction only in the $S$-state we can reduce Equation (1) to the following equation

$$T_L(f, k; k_0, z_3) = T^{(0)}$$

$$+ \frac{1}{(2\pi)^3} \int \frac{dk'}{k'} T_0(f, p_2(p), \bar{z}_2) T_L(p, k'; k_0, z_3)$$

$$\left[ z_3 - \frac{\hbar^2}{m} \left( p^2 + \frac{3}{4} k^2 \right) \right]$$

(3a)

where

$$y = \frac{p^2 - k^2 - k'^2/4}{k k'}$$

and $p_2(p) = \sqrt{p^2 + \frac{3}{4} k^2 - \frac{3}{4} k'^2}$.

(3b)

Here $L$ is the total angular momentum and $P_L(y)$ the corresponding Legendre polynomial. For the two-body $t$-matrix $t_0$ we have

$$t_0(k', k, z_2) = v_0(k', k) + \frac{1}{2\pi^2} \int \frac{dk''}{z_2} v(k', k'') t_0(k'', k, z_2)$$

(4)

The inhomogeneous term $T^{(0)}$ will not be specified in detail, since in what follows we will deal with the homogeneous equation only.

We restrict ourselves to separable potentials of the form

$$v_0(k', k) = -\lambda g(k') g(k)$$

(5)

Then we have for the $t$-matrix

$$t_0(k', k, z_2) = g(k') g(k) \tau(\sqrt{z_2})$$

(6a)

where

$$\tau(\sqrt{z_2}) = -\left( \lambda^{-1} + \frac{1}{2\pi^2} \int \frac{dk'}{z_2} \frac{g^2(k')}{z_2 - \frac{\hbar^2}{m} k'^2} \right)^{-1}$$

(6b)

The condition $\text{Im}(z_2)^{1/2} > 0$ defines the physical sheet of the two-particle $t$-matrix.

Inserting Equation (6) into (3a) we get

$$T_L(f, k; k_0, z_3) = g(f) \tau(\sqrt{z_2}) F_L(k, z_3)$$

(7)