Description of Negative Parity Yrast States in $^{156}$Er

M. Ploszajczak
Institut für Kernphysik, Kernforschungsanlage Jülich, Jülich, West Germany

Amand Faessler
Physics Department, State University of New York Stony Brook, N.Y., USA

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Positive and negative parity yrast states are studied in $^{156}$Er with a particle number projected Hartree-Fock-Bogoliubov method constrained on an average angular momentum. The theory predicts a second anomaly of the positive parity yrast states due to the alignment of a $h_{11/2}$ proton pair. The double backbending in the negative yrast band is understood: At $J^\pi = 9^-$ to $11^-$ it is due to the intersection of the $(\pi g_7/2^+ \times \pi h_{11/2}^2/2^-)$ and the $(\nu i_{13/2}^3/2^+) \times (\nu h_{9/2}^3/2^-)$ 2 q.p. bands. The second backbending found experimentally from $J^\pi = 21^-$ to $23^-$ is connected with an alignment of a $i_{13/2}$ neutron pair of the core in the proton 2 q.p. band.

I. Introduction

The high spin, negative parity yrast sequences ($J^\pi = (2L - 1)^-$, $L = 1, 2, \ldots$) have been observed by many groups in several rare earth nuclei (see e.g. the compilation of Lieder and Ryde [1] and the references quoted therein). These states are often found to be situated close to the positive parity yrast states in the backbending region in which the moment of inertia increases suddenly. In this region, the gap between the positive parity, 0 q.p. yrast states and the 2 q.p. excitations vanish and those states mix strongly. At still higher angular momenta the yrast line is composed by the rotational band build on the 2 q.p. intrinsic excitation. The theoretical interest in the negative parity yrast states arises because of the intriguing question about the possible mechanisms which cause the anomalous behaviour of the moment of inertia at $J \approx 16 \hbar$ (in the rare earth region). Till now most of the theoretical works were devoted to the explanation of the ground state (g.s.) rotational band of even-even nuclei and the rotational bands in the odd-even systems. The studies seem to suggest that in the critical region, the decisive push for backbending in the whole rare-earth region is due to the breaking of the $i_{13/2}$ neutron pair and the alignment of them along the rotational axis. In other words, the 0-quasi-particle (0-q.p.) g.s. rotational band is replaced by the rotational band built on the 2 q.p. ($vi_{13/2}^3$) intrinsic state. The accurate description of backbending should in principle take into account the interaction between the intersecting bands of the same parity. This interaction may change significantly the q.p.-spectrum close to the Fermi surface. However, calculations which include this interaction require immense amounts of computing time and, therefore, they have not been done yet. In contrast, the negative parity states cannot mix with the positive parity zero q.p. band. Therefore, without going into the elaborate calculations, the studies of these bands in the same nucleus provide the opportunity for a better understanding of the phenomena occurring in the critical region. These are breaking of one pair of particles and alignment with the rotation axis, due to the Coriolis force (RAL) [2], breaking of all pairs (Mottelson-Valatin effect) [3], and gapless superconductivity [4]. The very convenient mathematical apparatus which allows for the simultaneous description of all these effects is given by the cranking Hartree-Fock-Bogoliubov theory (CHFB) (see [5] and [6] and references quoted therein). This theory has been extend-
ed to include the projection onto the correct number of particles by Faessler et al. for the studies of the even-even [6] and odd-even [7] nuclei. Here we will apply this approach for the studies of the negative parity rotational bands in the even-even nucleus. The bandheads are described as a negative parity 2q.p. excitation. As an effective two-body force we use the pairing and quadrupole-quadrupole \((P+QQ)\) force Hamiltonian of Kumar and Baranger [8].

The purpose of this work is to study in a microscopic model the behaviour of the negative— as well as the positive—parity bands in \(^{156}\)Er ([9–11]). We explain the backbending in the odd-spin, negative parity band observed in \(^{156}\)Er for the first time by the Brookhaven group [11]. Moreover, we perform the calculations of the yrast line above the first backbending at \(J \approx 12\ h\). At \(J =24\ h\) the calculated moment of inertia shows the second anomaly, namely it increases rapidly. A similar effect was recently reported by the Berkeley group in the neighbouring nucleus \(^{158}\)Er at \(J \approx 26\ h\) [12].

In Chapter 2 the most important formulas of the CHFB model are given and the construction of the negative parity bands is discussed. The reasons for backbending in the negative parity band of \(^{156}\)Er as well as the description of the positive parity yrast line is given in Chapter 3. Finally, in Chapter 4 we summarize the main results.

\section{Theory}

The model Hamiltonian is:

\[ H_\text{FB} = \sum_{\alpha} (\mathcal{V}_\alpha + \mathcal{H}_\alpha) + \sum_{\alpha, \beta} \mathcal{W}_{\alpha, \beta} c^\dagger_{\alpha, \beta} c_{\alpha, \beta} \]

\[ - \omega_c \sum_{\alpha, \beta} \langle \alpha | j_x | \beta \rangle c^\dagger_{\alpha} c_{\beta} \]

\[ \mathcal{V}_\alpha = (\mathcal{V}_{\alpha} + \mathcal{H}_\alpha) = \sum_{\alpha, \beta} \mathcal{V}_{\alpha, \beta} c^\dagger_{\alpha, \beta} c_{\alpha, \beta} \]

\[ \mathcal{W}_{\alpha, \beta} = \sum_{\alpha, \beta} \mathcal{W}_{\alpha, \beta} c^\dagger_{\alpha} c_{\beta} \]

\[ \mathcal{H}_\alpha = \sum_{\alpha, \beta} \mathcal{H}_{\alpha, \beta} c^\dagger_{\alpha} c_{\beta} \]

where \(|\alpha, \beta\rangle\) are the single particle (s.p.) basis states and the s.p. Hamiltonian is:

\[ \langle \alpha | h | \beta \rangle = \delta_{\alpha, \beta} - \omega_c \sum_{\alpha, \beta} \langle \alpha | j_x | \beta \rangle c^\dagger_{\alpha} c_{\beta} \]

\[ \langle \alpha | h | \beta \rangle = -\omega_c \sum_{\alpha, \beta} \langle \alpha | j_x | \beta \rangle c^\dagger_{\alpha} c_{\beta} \]

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\[ \langle \alpha | h | \beta \rangle = \delta_{\alpha, \beta} - \omega_c \sum_{\alpha, \beta} \langle \alpha | j_x | \beta \rangle c^\dagger_{\alpha} c_{\beta} \]

The spherical s.p. energies entering (2) are taken from [8]. Also the oscillator energy \(\hbar \omega_0 = 41.2 \cdot A^{-1/3} \) MeV as well as the parameters \(\omega_c = (2 N_c / A)^{1/3}\) are the same as in [8].

The effective, short range, nucleon-nucleon interaction is represented by the second term in (1). There \(G_c\) and \(G_n\) are the pairing constants for protons and neutrons respectively. The angular momentum \(J\) and the particle number \(N\) are conserved in average by adjusting the corresponding Lagrange multipliers \(\lambda_i, \omega\) in the equations:

\[ \langle \Psi_{\text{HFB}} | \hat{N}_i | \Psi_{\text{HFB}} \rangle = N_i \]

\[ \langle \Psi_{\text{HFB}} | J_z | \Psi_{\text{HFB}} \rangle = J(J+1) - \langle J^2 \rangle. \]

The wave function \(\Psi_{\text{HFB}}\) is defined as the quasi-particle vacuum and the quasi-particle creation and annihilation operators are given by:

\[ a^\dagger_{\alpha} = \sum_{\alpha} (A_{\alpha} c^\dagger_{\alpha} + B_{\alpha} c_{\alpha}) \]

\[ a_{\alpha} = \sum_{\alpha} (B^*_{\alpha} c^\dagger_{\alpha} + A^*_{\alpha} c_{\alpha}) \]

Minimizing the expectation value of the model Hamiltonian with the wave function \(\Psi_{\text{HFB}}\), and neglecting the q.p. interaction we obtain the so-called Hartree-Fock-Bogoliubov (HFB) equations:

\[ \begin{pmatrix} \hbar - \Delta \chi \hbar \end{pmatrix} \begin{pmatrix} A^* \nabla \nabla \end{pmatrix} = E \begin{pmatrix} A^* \nabla \nabla \end{pmatrix} \]

where:

\[ \langle A^* \nabla \nabla \rangle = \langle \hat{A} \hat{A} \rangle = -\Delta_{\alpha} \delta_{\alpha, \beta} \]

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It is convenient to introduce in (5) the density and pairing matrices. They are defined as:

\[ \rho_{\alpha, \beta} = \langle \Psi_{\text{HFB}} | c^\dagger_{\alpha} c_{\beta} | \Psi_{\text{HFB}} \rangle = \langle \hat{B}^* \hat{B} \rangle_{\alpha, \beta} \]

\[ \kappa_{\alpha, \beta} = \langle \Psi_{\text{HFB}} | c^\dagger_{\alpha} c_{\beta} | \Psi_{\text{HFB}} \rangle = \langle \hat{B}^* \hat{A} \rangle_{\alpha, \beta} \]

In solving Equations (5) we take advantage of the symmetries of the HFB wave function obtained for the general Hamiltonian in the rotating system. Since the isospin and parity symmetries are conserved for the Hamiltonian \(H' = H - \omega_0 j_z\), the solutions of (5) can be independently found in the four square submatrices which have no common matrix elements. Moreover, \(H'\) obeys the symmetry under the rotations through an angle \(\pi\) around the intrinsic axis \((e^{i j_x})\). Therefore, by choosing as the basic functions the eigenfunctions of this symmetry operator we may further reduce the dimension of the HFB submatrices. Those eigenstates of \(e^{i j_x}\) are given by:

\[ |\alpha \rangle = \frac{1}{\sqrt{2}} (|\alpha \rangle + P_0 |\alpha \rangle) \]

\[ |\alpha \rangle = \frac{1}{\sqrt{2}} (|\alpha \rangle - P_0 |\alpha \rangle) \]

with the phase factor \(P_0 = (-1)^{I_s - I_p + 1/2}\). The spherical shell model states entering (7) are characterized by the