Theory of Pion-Nucleus Scattering Lengths

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We calculate the scattering lengths for pions on light nuclei, in a multiple scattering theory given earlier. We include all effects through second order in the pion-nucleon scattering amplitudes, using a simple nuclear model. We show how to calculate the binding corrections (BC) consistently, and find large cancellations between different BC terms. The resulting scattering lengths are smaller than those obtained from measurements of pi-atomic level shifts. The size of the difference is consistent with the contribution of pion absorption, which we estimate.

In an earlier paper (denoted here as I) Moyer and Koltun presented a theory for pion-nucleus scattering lengths based on multiple scattering of the pion on nucleons in the target. (See also Ref. 2.) In the present paper we update and improve that work. The major part of this work is devoted to a consistent treatment of the effects due to pion scattering from bound nucleons. In [I] the binding correction was found to give a dominant contribution to the π-nucleus scattering length. We show in what way the calculation of the binding correction in [I] was not consistent. Our new calculation gives a smaller, but important binding correction to the impulse approximation for isospin zero nuclei. We calculate the scattering lengths for several light nuclei, using a newer analysis [3] of the low energy pion nucleon scattering data. Recent pionic atom data [4] has provided a more accurate and more complete set of ls level shifts and widths from which "experimental" scattering lengths may be obtained. The latter analysis has been done by Hüfner et al. [5]. These authors have also shown that the scattering lengths may be obtained from an x-particle model of the nuclear targets, using π-α scattering as the elementary process in a multiple scattering series. Our results show that even from the elementary πN scattering one may be successful in reproducing the experimental results.

The theory of [I] follows the method of Watson, in which the π-nucleus scattering amplitude T is expanded in terms of the free π-N amplitude t(j) for the j-th nucleon. As in [I] we write the multiple scattering series in the form

\[ T = T_{ss} + \Delta T_2 + \Delta T_B + \Delta T_{ms} \]  
\[ T_{ss} = \sum_{j=1}^{N} t(j) \]  
\[ \Delta T_2 = \sum_{j \neq l}^{N} t(j) G t(l) \]  
\[ \Delta T_B = \sum_{j}^{N} t(j) [G - g(j)] t(j). \]

G is the propagator for the pion in the nucleus, g(j) the propagator for a free π - N system. T_{ss} is the single scattering term in the impulse approximation, \( \Delta T_2 \) represents double scattering from different nucleons, and \( \Delta T_B \) is the term for double scattering on the same bound nucleon. The contribution of all higher orders is included in \( \Delta T_{ms} \).

The propagator G is a many-body operator, which includes excitation of the target nucleus, as well as propagation of the pion. We consider a single-particle model of the target. Then, we evaluate (1b) and (1c), ignoring the Pauli Principle for intermediate nuclear states. As discussed in [I], this choice has no effect on the sum of the terms \( \Delta T_2 + \Delta T_B \), as long as the ground state is appropriately antisymmetric, but it does simplify the evaluation of each term. In a single-particle model, without requiring the Pauli Principle,
the binding correction term (1c) can be evaluated as a
restricted three-body problem, of the pion scattering
from a single nucleon bound in a potential. Also, the
double scattering term (1b) takes a particularly simple
form (see Eq. 5) since excited nuclear states do not
contribute to $\Delta T_2$, in this model. The only approxima-
tion in this approach is the use of a single-particle
model; the particular treatment of the Pauli Principle
is exact as long as all terms through a given order in
$r(j)$ are retained.
The $\pi$-nucleus scattering length is given by

$$a(i_3, Z, N) = -(\mu/2\pi) g^{-1}(A) \langle T \rangle$$

where $\langle T \rangle$ is the expectation value of (1) in the target
$(Z, N)$ ground state, for a pion of charge $i_3$. The pion
mass is $\mu$, and $g(A) = (1 + \mu / AM)$ is a reduced-mass
factor for a nucleus of mass $AM$; $M$ is the nucleon
mass, and $A = Z + N$.

As in [1], we use a scattering length approximation
for the $s$-wave $\pi-N$ amplitudes. In terms of the
scattering lengths for $\pi-N$ isospin 1/2, 3/2, we use
the isoscalar length $a_0 = 1/3(a_{1/2} - 2a_{3/2})$, and
the isovector length $a_1 = (a_{3/2} - a_{1/2})/3$. Corrections to the
scattering length approximation induced by a (small)
finite range to the $\pi-N$ interaction, have been found
to be small by Moyer [2] and by Myhrer and Silbar
[6].

In [1], a correction for $p$-wave $\pi-N$ scattering due
to nucleon motion in the target (recoil) was included
in the calculation of $T_{ss}$. Moyer later estimated [2] the
$p$-wave contribution to $T_2$; we shall include this as
well.
The binding correction $\Delta T_B$ was discussed and
calculated in [1]. However, we have discovered that
the approximation used in [1] is not consistent with
the use of the impulse approximation for $T_{ss}$ in (1a),
to second order in $a_{\pi N}$. The binding correction depends
on the difference between the bound and free prop-
grators, $G$ and $g$, respectively (suppressing the particle
label, $j$). In a single particle model, in which the target nucleon has a binding energy $E_j$, in a potential $U$, we
may write (see (2.3), (2.8))

$$G = (\varepsilon_\pi + E_j - K_\pi - K - U + i\eta)^{-1}$$

$$g = (E_0 - K_\pi - K + i\eta)^{-1}$$

where $K_\pi$ and $K$ are the kinetic energy operators of the
pion and the nucleon, respectively, and $\varepsilon_\pi$ is the kinetic
energy of the pion. The energy $E_0$ is in some sense
arbitrary, but should be chosen to have the same value in
evaluating both $T_{ss}$ and $T_B$.

The scattering length approximation for $T_{ss}$ amounts
to evaluating the free $\pi - N$ amplitude $t$ in (1a) on
shell, so that

$$E_0 = \varepsilon_\pi + E_k$$

where $E_k$ is the kinetic energy of the struck nucleon,
suitably averaged over the momentum distribution
in the target. For this choice of $E_0$, it follows from (3) that

$$G^{-1} = g^{-1} - U + E_\Lambda - E_k$$

The difference $(E_k - E_\Lambda)$ was neglected in [1]; this
leads to an overestimate of the binding effect, since
$E_k - E_\Lambda > 0$, while $\langle U \rangle < 0$. (For the target ground
state $\langle E_\pi - E_k - U \rangle = 0$.) With the choice (4a) for the
energy $E_0$, the single scattering term (1a) is properly
evaluated in the impulse approximation.

Alternatively, one may calculate the binding correction
with a different choice of the parameter $E_0$, given by

$$E_0 = \varepsilon_\pi + E_\Lambda$$

(This is the choice we shall actually use for calculation.)

With this choice of $E_0$, we obtain from (3) the following
relation between the bound and free propagators:

$$G^{-1} = g^{-1} - U$$

This is the form which was used in [1] for the evaluation
of $T_B$ (see 1: (5.3), (5.5)).

However, now the free $\pi-N$ amplitude must be
evaluated with the same choice of $E_0$, given by (5a),
which is off the energy shell by the amount $E_\pi - E_k$.
This difference induces a correction to the impulse
approximation (used in Eq. (1a)) as was first noticed
by Myhrer and Silbar [6], who showed the connection
of this effect to unitarity.

Therefore, if we choose $E_0$ as in (5a), the total correc-
tion to the impulse approximation due to the binding
of the target nucleon is given by the sum of $\Delta T_B$ (1c)
evaluated using Equation (5b), and the "unitarity
correction" just mentioned. To second order in $a_{\pi N}$,
this sum is the same as the value of $\Delta T_B$ obtained using
Equations (4), with the other choice of $E_0$, and with
no correction to (1a). In (1)*, they chose $E_0$ as in (5),
but omitted the "unitarity correction", which term,
as we shall see, considerably reduces the total binding
correction.

We have now recalculated the total binding correction with $E_0$
chosen as in (5), to second order, properly
including the unitarity correction. Our calculation of
$\Delta T_B$ closely follows that of [1], and the unitarity
correction is calculated, as in [6], using the nuclear
model parameters of [1]. In the numerical results for
the scattering lengths (Table I) we refer to the sum of
both contributions as $a_B$.

* The numerical results for the binding correction in [1] were
presented with the wrong sign. With the correct sign it will partly
cancel the "unitarity correction" of [6].