Hadronic Jets Associated with a Photon Trigger

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Abstract. We consider, within Quantum Chromodynamics, the fragmentation function of a quark into a hadron and a photon. A simple analytic expression is given which is adequate for practical applications. A measurable quantity is proposed, which is independent of the hadronization process and tests the underlying dynamics.

Due to the point-like nature of the photon, the structure functions of the photon are entirely calculable within Quantum Chromodynamics [1]. The distribution function of a quark (or gluon) in a photon can be studied in the photoproduction experiments [2], while the fragmentation function of a quark into a photon can be studied at machines like PETRA [3]. A simple parametrization of these functions can be found in [4]. More informations can be extracted by examining multiparticle spectra, namely the distribution of a quark pair within a photon [5] or the fragmentation function of a quark into a quark and a photon [6]. We examine here in detail the latter case.

Consider a highly virtual quark of invariant mass $Q$ which by emitting gluons degrades down to invariant mass $p$, emits a photon and by further gluonic emission ends up into a quark of mass $Q_0$. Following this picture [7] the fragmentation function $f_{q/q}(x_1, x_2, Q^2, Q_0^2)$ of a quark into a quark and a photon carrying fractions of momentum $x_1$ and $x_2$ respectively is given in the valence approximation* by

$$f_{q/q}(x_1, x_2, Q^2, Q_0^2) = \frac{Q^2}{Q_0^2} \frac{dp^2}{p^2} \int dx_1 dx_2 x_1^{n_1-1} x_2^{n_2-1} \phi(x_1, x_2)$$

where

$$\phi(z) = \frac{2\pi e^2}{z(1-z)}$$

Defining the double moments

$$\phi(n_1, n_2) = \int dx_1 dx_2 x_1^{n_1-1} x_2^{n_2-1} \phi(x_1, x_2)$$

we immediately obtain

$$f_{q/q}(n_1, n_2, Q^2, Q_0^2) = \frac{Q^2}{Q_0^2} \int \frac{dp^2}{p^2} f_{q/q}(n_1 + n_2 - 1, Q^2, p^2) \phi(n_1, n_2)$$

from which

$$\phi(n_1, n_2) = \int dz z^{n_1-1} (1-z)^{n_2-1} \phi(z)$$

Using also

$$f_{q/q}(n_1, Q^2, p^2) = \left[ \frac{\phi(Q^2)}{\phi(p^2)} \right]^{(d)}$$

we find after integration

$$f_{q/q}(n_1, n_2, Q^2, Q_0^2) = \frac{\alpha}{2\pi e^2} \ln \frac{Q^2}{A^2} (B(n_1, n_2-1) + B(n_1+2, n_2-1))$$

where

$$B(n_1, n_2) = \frac{1}{1 + d(n_1 + n_2 - 1) - d(n_1) - d(n_2)}$$

The term in the square brackets represents the QCD correction to the Born term. For consistency check we can observe the following: integrating $f_{q/q}(x_1, x_2, Q^2, Q_0^2)$ over $x_1$ we should find the photon distribution within a quark. Indeed, by putting $n_1 = 1$ in (7), we recover, ignoring subleading logarithms, the valence piece of $f_{q/q}$ [4].

To invert the double moments, a generalization

* We work within the valence approximation, because we are interested in the large $x_1 + x_2$ region where the valence term dominates.
of Yndurain's method [8] has been proposed [9]. However this method is not accurate. Rather we can obtain an analytic expression which is exact in the large $x_1 + x_2$ region. This is not a drawback, since from kinematical considerations the fragmentation functions are explored mostly in the large $x$ region. When $x_1 + x_2 \to 1$ we can substitute in (1) the large $x$ approximation for $f_{q/q}(x, Q^2, k^2)$

$$
\frac{f_{q/q}(x, Q^2, k^2)}{f_{q/q}(x, Q^2, k^2)} = \exp(\frac{BT}{T(A T)}(1 - x)A^{-1}T - 1)
$$

where $T = \ln \left[ \frac{a_1(Q_0^2)}{a_1(Q_2^2)} \right]$ and $A, B$ are constants [4]. We find

$$
\frac{f_{q/q}(x_1, x_2, Q^2, Q_0^2)}{f_{q/q}(x_1, x_2, Q^2, Q_0^2)} = \frac{\alpha}{2\pi^2} \ln \left[ \frac{Q^2 Q_0^2}{Q^2 Q_0^2} \right] \frac{1}{A^2 1 - x_1 + A \ln 1 - x_2}
$$

with $T = \ln \left[ \frac{a_1(Q_0^2)}{a_1(Q_2^2)} \right]$. In the limit $T \to 0$ the term in the square bracket gives $\delta(1 - x_1 - x_2)$. Figure 1 shows $f_{q/q}$ for the $u$ quark, as given by (9), for two values of $Q^2$ ($Q = 30, 200$ GeV) and using $Q_0^2 = 3$ GeV$^2$.

To obtain the hadron–photon distribution within a quark we have to convolute $f_{q/q}(x_1, x_2, Q^2, Q_0^2)$ with the distribution $f_{h/q}(x, Q_0^2)$ which is extracted from experiment. Parametrizing

$$
f_{h/q}(x, Q_0^2) = c(1 - x)
$$

it is easy to find

$$
\frac{d\sigma^{\pi^0\gamma}}{d^2x} = \frac{1}{2\pi^2} \frac{\alpha}{c} \ln \left[ \frac{Q^2 Q_0^2}{Q^2 Q_0^2} \right] \frac{1}{A^2 1 - x_1 + A \ln 1 - x_2}
$$

Figure 2 shows the above function for the $u$ quark and we have used $c = 0.3$.

At the electron–positron storage rings we can measure separately the $\pi^0\gamma$ and $\pi^0$ inclusive spectra. Then the following quantity

$$
R(x_1, x_2) = \frac{d\sigma^{\pi^0\gamma}}{d^2x} \frac{d\sigma^{\pi^0}}{d^2x}
$$

is somehow stabilized and it is in rough agreement with (9) for $x > 0.3$.