Non-Gaussian corrections to higgs mass in autonomous $\lambda \phi^4_{3+1}$

Uwe Ritschel
Fachbereich Physik, Universität GH Essen, D-45117 Essen, Germany
Received: 28 December 1993

Abstract. Recent calculations in one-loop and Gaussian approximation, using the so-called autonomous renormalization scheme, indicate a comparatively massive, narrow Higgs excitation at about 2 TeV. Here I show that this result persists in the framework of a post-Gaussian variational approximation for the pure $O(N)$-symmetric $\phi^4$-theory for $N > 1$. The method is based on nonlinear transformations of path-integral variables, and the optimization amounts to a Schwinger-Dyson-type summation of diagrams. In the case of $O(4)$, for example, I find $M_{\text{Higgs}} = 2.3$ TeV, compared with 1.9 TeV and 2.1 TeV in one-loop and Gaussian approximation, respectively. My results are also consistent with the Consoli-Stevenson conjecture that the Gaussian and one-loop results for $N = 1$ are exact and may even hold for general $N$.

1 Introduction

Triviality of the $\phi^4$-theory in four dimensions emerges as a simple consequence of renormalization-group (RG) improved perturbation theory [1], and, during the years, it has been corroborated by a variety of field-theoretical methods [2]. While the phenomenon is the foundation of the e-expansion, the most elegant and efficient computational tool in critical phenomena, it has been a nuisance in particle physics, because it seems to destroy the Higgs mechanism. One way out of this dilemma is to introduce some finite but possibly large cutoff and thus regard the $\phi^4$ model as an effective field theory. Alternatively, it was attempted to save the model as a renormalizable field theory by employing non-perturbative methods. Following this direction, Stevenson and Tarrach [3] suggested certain UV-flows for the bare parameters that led to a non-trivial Gaussian effective potential (GEP) [4]. The method was termed autonomous renormalization (AR) because it is not related to the perturbative renormalization, the essential novelty being an infinite wavefunction renormalization.

Recently essential progress has been made in understanding the theoretical basis of the AR and in deriving predictions for physical quantities from it. In a series of papers Consoli et al. [5] found that the AR can be applied also in the one-loop approximation and that the UV-flows for the bare parameters can be systematically derived from a RG analysis [6]. Further, they discovered that definite predictions for the Higgs mass can be obtained by imposing appropriate normalization conditions on the effective potential and under the natural assumption that the model has massless particles in the symmetric phase. Compared with other methods, the AR predicts a relatively heavy Higgs particle at about 2 TeV [6, 7].

In the sequel Consoli and Stevenson were able to generalize the procedure to the effective action [8]. It turns out that field modes with finite momentum (FM) and, as a consequence, the particles of the model, are effectively non-interacting. The non-trivial structure of the effective potential, especially spontaneous symmetry breaking and the generation of the Higgs mass, is solely caused by the self-interaction of the constant mode. In most transparent form this can be derived in a system of finite size [9], where the non-perturbative treatment of the constant mode is conceptionally clear and straightforward.

Eventually, Consoli and Stevenson [8, 10] conjectured that the results for the effective potential and, in turn, the mass of the one-particle state in the asymmetric vacuum, the Higgs mass, may be even exact for $N = 1$. Their arguments are based on the observation that the effectively non-interacting FM modes can only contribute on the level of the two-point function and the vacuum functional. Thus, depending on the approximation chosen the relation between bare and renormalized mass parameter may change, but (after appropriate normalization) the renormalized quantities should be unchanged. This view is corroborated by the comparison between the one-loop and GEP result for $N = 1$. Compared with the one-loop effective potential, the Gaussian approximation takes into account an infinite subset of Feynman graphs - the well-known cactus diagrams - and the relations between bare and renormalized parameters are indeed different from the one-loop case. Nevertheless, both approximations yield the same Higgs mass. In fact, if the conjecture is correct, one should find in any approximation that is
compatible with effectively free particles the same result. A rigorous proof of exactness is however not available at the present moment.

The situation is more complicated in the case of general $N$. There one-loop and Gaussian approximation yield different results. The reason is most probably an inadequate treatment of the angular variables. So far the calculations were performed in "cartesian" coordinates, which leave the transversal (would-be) Goldstone modes with a non-vanishing mass. For this case Consoli and Stevenson conjectured that the Goldstone modes, due to their masslessness when treated correctly as collective variables, should not change the shape of the effective potential. As a consequence, for any value of $N$ the results for $N = 1$ should hold.

The main objective of the present paper is to examine whether the autonomous renormalization can be applied beyond one-loop and Gaussian approximation and thereby to check the validity of the Consoli-Stevenson conjectures. This will be achieved by taking into account a much larger class of Feynman diagrams in the framework of a post-Gaussian variational calculation, which nevertheless satisfies the requirement of being compatible with a free field theory. The procedure allows to generalize the variational approach in quantum field theory to non-Gaussian trial states in the canonical formalism [11, 12] and non-Gaussian trial actions in the path-integral formalism [13]. In the following I shall work in the covariant setting.

The rest of this paper is organized as follows: In Sect. 2 the variational approximation for the path integral is briefly described, in a form general enough to include non-Gaussian test actions. In Sect. 3 the AR is formulated for the Gaussian approximation in such a way that it can be easily generalized to more sophisticated variational methods. In Sect. 4 the non-Gaussian contributions to the effective potential are computed, and Sect. 5 is devoted to the renormalization of power divergences generated by the variational ansatz, which is the most difficult technical problem in the context of this type of variational approximation. In Sect. 6 the autonomously renormalized effective potential is calculated. Eventually, in Sect. 7 the results for the Higgs mass and other relevant parameters are discussed and compared with other approximations.

2 The variational approximation

The action considered throughout the paper is given by

$$S = \frac{1}{2} \int \left( \partial_{\mu} \phi^a \partial_{\mu} \phi^a + m_{\text{phys}}^2 \phi^a \phi^a \right) + \lambda \int \left( (\phi^a)^2 \right)^2,$$

with $\int \rightarrow \int d^4x$.

In order to approximate the effective potential, $V(\phi_c)$, I introduce the nonlinear transformation [12]

$$\bar{\phi}^4(p) = \tilde{\eta}^4(p) + \chi_0 \tilde{\delta}(p) + s \int c(q, r) \tilde{\eta}^4(q) \tilde{\eta}^4(r) \tilde{\delta}(p - q - r),$$

$$\bar{\phi}^a(p) = \tilde{\eta}^a(p) \quad a = 2, \ldots, N,$$

where $\int \rightarrow \int (2\pi)^d \delta(p) = (2\pi)^d \delta(p)$, and tildes indicate Fourier amplitudes.

Transformation (2.2) is appropriate for studying spontaneous symmetry breaking $O(N) \rightarrow O(N - 1)$. It was originally used in the canonical approach and is discussed in more detail in [12]. The ansatz is particularly well suited for a variational calculation, because it leaves the path-integral measure invariant (which simplifies the calculation) and the effective potential can be evaluated in closed form, i.e., it does not lead to a series expansion of the expectation value. The $c$-number $\chi_0$ and the correlation function $c(q, r)$ - the factor $s$ is split off for normalization and as a bookkeeping device - are variational parameters.

The upper bound on the effective potential derived from (2.2) is

$$V(\phi_c) \leq V_G(\phi_c) = \frac{1}{\int_s} \min \left\{ - \log N + N^{-1} \right\} \times \left\{ \int D\eta e^{-S_0[\eta]} \int_s \right\} \tilde{\phi}^4 = \text{fixed},$$

where the available variational parameters have to be optimized under the constraint

$$\phi^a_c = N^{-1} \int D\eta e^{-S_0[\eta]} \phi^a = \text{fixed},$$

i.e., with the expectation value of the field held fixed, and

$$N = \int D\eta e^{-S_0[\eta]}.$$

In the equations above the $\phi$'s have to be substituted with (2.2). $S_0$ is a quadratic test action, given by

$$S_0[\eta] = \frac{1}{2} \int \left\{ \eta^4(p) G_L^{-1}(p) \tilde{\eta}^4(-p) + \tilde{\eta}^4(p) G_T^{-1}(p) \tilde{\eta}^4(-p) \right\},$$

where $G_L$ and $G_T$ are the propagators for (with respect to the direction of symmetry breaking) longitudinal and transversal modes, respectively, which serve also as variational parameters. (In the Gaussian approximation to the effective potential the propagators are the only variational parameters.)

3 Autonomous Gaussian approximation

If the nonlinear term is absent in (2.2), the simple identity $\phi_c = \chi_0$ holds and the optimization yields the (covariant) GEP. (In contrast to the canonical formalism, this procedure can be straightforwardly extended to the effective action [13], but for my present purpose the effective potential is sufficient.) In the following I am summarizing the main results of the AR for the Gaussian approximation. This analysis was first done in the canonical formalism by Stevenson et al. [14]. In the next section the procedure will be extended to the non-Gaussian case by following essentially the same line of arguments.

The bare unoptimized GEP is given by

$$V_G = J^4 + (N - 1)J^T + \frac{1}{2} m_{\text{phys}}^2 [I^L + (N - 1)I^T + \phi_c^2]$$

$$+ \lambda \left[ 3(3 + 2 \phi_c^2) - 2 \phi_c^2 + 2(N - 1) \right] \times I^L (I^T + \phi_c^2) + (N^2 - 1)(I^T)^2,$$

(3.1)