Solid particle in the boundary layer of a rising bubble

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Abstract: The hydrodynamic interaction of a solid particle and the boundary layer around a rising bubble is analyzed in the before-contact state (BCS) of a flotation act. The lagging of the particle behind the basic outer flow is accounted for. The forces acting on the particle are qualitatively examined. A new term is introduced in the force balance - the migration force. An expression for the collision efficiency is proposed that concerns a particle already entrained in the bubble's boundary layer.

Key words: Flotation act; hydrodynamic interaction; boundary layer; collision efficiency

1. Introduction

The interaction of a solid particle and the hydrodynamic field of a rising bubble is of great importance for mineral processing since it determines the phenomena in the before-contact-state (BCS) of the flotation act [1]. The theoretical modelling of this process should comprise two basic moments:
1) the characterization of the flow field induced by the bubble, and
2) the elucidation of the forces acting on the particle, captured by this flow.

To characterize the peculiar features of the flow around a rising bubble, potential flow equations [2] or creeping flow equations [3] have been applied. There are also some papers which make use of numerical solutions of the Navier-Stokes equations for special cases (see for instance [5,4]).

As far as the particle’s behavior in this outer field is concerned, it is usually regarded as an inertialless sphere; the only attempt to take account of inertial effects is in [6].

The results of the model flotation experiments show that the optimum size of the bubbles is in the range of $5 \times 10^{-4} - 10^{-3} \text{ m}$ [7]. The average rising velocities of such bubbles are $5 \times 10^{-2} - 20 \times 10^{-2} \text{ m/s}$ [7]. Under these circumstances the fluid flow induced by the bubble may be regarded as potential (inviscid) except for a thin layer in the immediate vicinity of it, where the viscous effects cannot be neglected. This region is known as a boundary layer with a thickness $\delta \approx R_b/\sqrt{Re_b} \approx 10^{-4} \text{ m}$. Here $Re_b = V_b R_b/\nu$ is the Reynolds’ number of the flow ($V_b$ - velocity, $R_b$ - radius of the bubble, $\nu$ - kinematic viscosity coefficient). Hence, the existence of the boundary layer should not be ignored for particles of the size $R_p \leq \delta$. If such particles are captured by the boundary layer additional interaction phenomena appear which have not been considered until now, namely:
1) Faxen’s deformation velocity [8]; and
2) Saffman’s lateral migration forces [9].

The aim of the present paper is to analyze the forces acting on a particle entrapped in the boundary layer of a rising bubble, taking into consideration the above-mentioned effects.

2. Interaction of a particle with an outer hydrodynamic field

A spherical bubble having a radius $R_b$ rises with a velocity $V_b$ in a fluid with viscosity $\mu$ and density $\rho_f$. Figure 1 shows the spherical coordinate system $(r, \varphi, \theta)$ originating at the bubble’s center 0. The hydrodynamic field around the bubble is modelled by Moore’s solution [10]; here only its boundary layer part is cited.
For BCS of the flotation act it is sufficient to know the details of the fluid flow at the frontal hemisphere of the rising bubble, which is practically free of surfactant molecules. Therefore, Moore’s solution gives an adequate modelling of the flow field in this case. The boundary layer region around the frontal hemisphere of the bubble is the outer hydrodynamic field, which entraps the solid particle during the flotation act.

The recent theoretical treatments of the interaction of a particle and a flow field may be summarized as follows:

1) A neutrally buoyant particle (i.e., $\Delta \rho = \rho_p - \rho_f = 0$). A locally confined hydrodynamic disturbance emerges in the basic outer flow. The local Reynolds number in the vicinity of the particle is low and the disturbance field has a Stokesian character ($Re_L = \frac{R_p}{\nu}$, $\nu$ is the undisturbed velocity, $R_p$ is the sphere’s radius). As Faxen [8] has shown, the result is the lagging of the particle behind the undisturbed fluid velocity at the same place. This deformation velocity is given by

$$v_L = \frac{1}{6} \frac{R_p^2}{\nu} \nabla^2 \bar{v}$$

For larger particles in addition to the viscous interactions, inertia factors also become important. Saffman [9] has established that this leads to the appearance of the so-called lift forces, which cause lateral migration with regard to the main flow

$$F_L = 6.46 \mu R_p V_L \sqrt{Re_L}$$

Here $V_L$ is the lag velocity of the particle, $Re_L$ is the local Reynolds number in the vicinity of the solid sphere; Saffmann did not specify this velocity. For locally confined hydrodynamic disturbances, however, one possibility is the Faxen’s deformation lagging, i.e., $v_L - v_f$. Another possibility is a non-neutrally buoyant particle, ($\Delta \rho \neq 0$); it possesses a sedimentation velocity in immobile fluid that becomes a source of additional disturbance of the main outer flow. This effect couples with the above-mentioned [12, 11].

For both kinds of particles the presence of an interface in their vicinity results in additional deformation of the hydrodynamic field. Therefore, the proximity of the bubble’s interface should play a significant role.