Semileptonic decays of $B$ mesons into excited charm mesons: leading order and $1/m_c$ contributions

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Abstract. We use the heavy quark effective theory to investigate the form factors that describe the semileptonic decays of a $B$ meson into excited daughter mesons. For an excited daughter meson with charm, a single form factor is needed at leading order, while five form factors and two dimensionful constants are needed to order $1/m_c$ in the heavy quark expansion. For non-charmed final states, a total of four form factors are needed at leading order. For the process $B\rightarrow D^{(*)}X\ell\nu$, four form factors are also needed at leading order.

I Introduction

The heavy quark effective theory (HQET) \cite{1} has been a very useful tool in the study of electroweak decays of heavy hadrons. It has been used to find relationships among the form factors describing the weak decays of heavy mesons, namely $B\rightarrow D$ and $B\rightarrow D^*$. It has also been applied to decays of heavy baryons \cite{2} to leading order in the heavy quark Lagrangian, as well as to $1/m_c$ correction terms in the decays of both heavy mesons and baryons \cite{3}.

In addition to these exclusive processes into the so-called elastic channels, Isgur and Wise \cite{4} have studied decays into 'inelastic' channels, and have discussed their results in terms of a Bjorken sum rule \cite{5}, which has also been developed using the techniques of HQET. More recently, Falk \cite{6} has developed representations of states of arbitrary spin.

In this paper, we consider another set of electroweak processes, namely the decay of the $B$ meson into any of the allowed excited charm daughter mesons. The motivation here is that the $B$ can decay weakly, but because the phase space available is sizeable, decays to excited daughter mesons (with the subsequent decays into multihadron final states) may provide a large contribution to the total rate of the $B$.

In fact, about twenty-three percent of charged $B$ meson decays are to the inclusive channel $B\rightarrow \ell\nu + \text{hadrons}$ \cite{7}. In addition, recent experiments indicate that the $D$ and $D^*$ channels, the so-called elastic channels, saturate the $B$ semileptonic decays (to charm) to only about seventy percent \cite{8}. This is evidence that semileptonic decays to excited mesons form a significant fraction of the total semileptonic decay rate of the $B$ meson. Undoubtedly, decays to the lower lying excited mesons, such as those already treated by Isgur and Wise \cite{4}, will provide the major contribution to the inelastic channels. Nevertheless, in the event that semileptonic decays to even more highly excited states are observed, we enumerate the form factors necessary to describe such decays.

In addition to these decays, there is some interest in decays of the type $B\rightarrow D^{(*)}X\ell\nu$. In particular, if $X$ is one or two pions, this costs relatively little in phase space, and may receive both resonant and non-resonant contributions. It may therefore be preferable to treat these decays on a separate footing from the decays to charm resonances.

The corresponding decays of the $D$ meson are, of course, more difficult to treat. Experimentally, the situation is probably more promising than in the decays of the $B$ meson, since there is no expectation that the $K$ and $K^*$ channels should saturate the semileptonic decays of this meson. For this reason it may be even more interesting to examine semileptonic decays of $D$ mesons to excited final states, despite the apparent theoretical difficulty.

In the next section of this note, we briefly discuss the representations of the states in which we are interested. We treat the semileptonic decays of the $B$ meson to excited charm mesons to leading order, as well as the semileptonic decays of the $D$ meson to excited mesons, and $B\rightarrow D^{(*)}X\ell\nu$, in Sect. III. In Sect. IV, we include $1/m_c$ terms in the semileptonic decays of the $B$ meson to excited charm mesons, and in Sect. V we present our conclusions.

II Representation of $D^{(j,\ell)}$ states

An excited $D$ meson with total angular momentum $J$ will, in general, be represented by an object linear in a polarisation tensor, $\eta_{\mu_1\cdots\mu_s}(v)$. This polarization tensor
is symmetric, transverse and traceless. The latter two properties are expressed by

\[ v_{\mu_1} \cdots \cdot \mu_k(v) = 0, \quad g_{\mu_1 \mu_2} \eta^{\mu_3 \cdots \cdot \mu_k(v)} = 0. \] (1)

For a state consisting of a heavy quark \( Q \) and a light component with the quantum numbers of an antiquark, the specific representation of any particular state will depend on the angular momentum \( j \) of the light component (antiquark) of the state. It is thus more convenient to refer to \( j \) than to \( J \), since there will be a degenerate doublet of states with \( J = j_1/2 \).

The parity of such a state is determined by the orbital angular momentum \( \ell = j - 1/2 \) between the light component and the heavy quark. For states with \( \ell = j + 1/2 \), Falk [6] writes the representations as

\[
\begin{align*}
V_{\ell_1 \cdots \ell_k} &= \frac{1}{\sqrt{2}} A_+ \eta^{\ell_1 \cdots \ell_k + 1} \gamma_{\ell_k + 1}, \\
P_{\ell_1 \cdots \ell_k} &= \left( \frac{2k + 1}{2k + 2} \right) A_+ \eta^{\ell_1 \cdots \ell_k} \left[ \delta_{\ell_1} \cdots \delta_{\ell_k} \\
&\quad - \frac{1}{2j + 1} A_{\ell_1 \cdots \ell_k} \right] \gamma_5, \\
A_{\ell_1 \cdots \ell_k} &= \left( \frac{2k + 1}{2k + 2} \right) \eta_{\ell_1 \cdots \ell_k} \\
&\quad - \frac{1}{2j + 1} A_{\ell_1 \cdots \ell_k} \gamma_5,
\end{align*}
\] (2)

In the above, \( k = j - 1/2 \), and the state \( \ell = j + 1/2 \) \( = \ell + 1 \), while \( \ell = j - 1/2 = \ell \). These two states are degenerate at leading order in HQET.

For light component with \( \ell = j + 1/2 \), the two possible states have the same values of \( J \) \( (J = j + 1/2) \) as the states described above. They are the counterparts of the \( \ell \) \( (P, V) \) doublet of states differs by unity from \( \ell \) for the \( (B, S) \) doublet of states with the same \( J \). Thus, one may write

\[
\begin{align*}
B_{\mu_1 \cdots \mu_k} &= \frac{1}{\sqrt{2}} A_+ \eta^{\mu_1 \cdots \mu_k + 1} \gamma_{\mu_k + 1}, \\
S_{\mu_1 \cdots \mu_k} &= \left( \frac{2k + 1}{2k + 2} \right) A_+ \eta^{\mu_1 \cdots \mu_k} \left[ \delta_{\mu_1} \cdots \delta_{\mu_k} \\
&\quad - \frac{1}{2j + 1} A_{\mu_1 \cdots \mu_k} \right] \gamma_5,
\end{align*}
\] (3)

with \( A_{\mu_1 \cdots \mu_k} \) as defined above. We emphasize that, to leading order in HQET, the \( (P, V) \) states form a degenerate doublet, as do the \( (B, S) \) states, but the two doublets will, in general, have different masses. Note that for all four of these states, denoted \( X_{\mu_1 \cdots \mu_k} \),

\[
\begin{align*}
v_{\mu_1} X_{\mu_1 \cdots \mu_k} &= 0, \\
g_{\mu_1 \mu_2} X_{\mu_3 \cdots \mu_k} \gamma_{\mu_1} &= 0, \\
\gamma_5 X_{\mu_1 \cdots \mu_k} &= X_{\mu_1 \cdots \mu_k} \gamma_{\mu_1}, \\
\gamma_5^2 X_{\mu_1 \cdots \mu_k} &= -X_{\mu_1 \cdots \mu_k},
\end{align*}
\] (4)

In the last of these equations, the negative sign occurs for the \( (P, V) \) doublets of states, while the positive sign occurs for the \( (B, S) \) doublets of states. Further details of the structure and properties of these states are given in Falk’s article [6].

### III \( B \rightarrow D^{(J', \ell')} \): Leading order form factors

To leading order in HQET, we are interested in the matrix element

\[
\mathcal{A} = \langle D^{(J', \ell')}(v') | h^{(0)}_0 | B(v) \rangle \] (5)

Representing the \( B \) meson by

\[
B(v) \to \frac{1}{\sqrt{2}} A_+ \gamma_5 \equiv M_B(v), \] (6)

representing any of the states discussed in the previous section by \( M^{(0)}_{\ell_1 \cdots \ell_k} (v') \), and using HQET, we may write this matrix element as

\[
\langle D^{(J', \ell')}(v') | h^{(0)}_0 | B(v) \rangle = \text{Tr} \left[ R_{\mu_1 \cdots \mu_k} M^{(0)}_{\ell_1 \cdots \ell_k} (v') \right]. \] (7)

Here \( R_{\mu_1 \cdots \mu_k} \) is the most general tensor that we can build. Clearly, \( R_{\mu_1 \cdots \mu_k} \) may contain no factors of \( \gamma_5 \), nor of \( \gamma^{\mu_1 \cdots \mu_k} \), nor \( \gamma_5 \). One is left with

\[
R_{\mu_1 \cdots \mu_k} = v_{\mu_1} \cdots v_{\mu_k} \left[ \xi^{(J', \ell')}(v \cdot v') + \xi^{(J', \ell')}(v \cdot v') \gamma_5' \\
+ \xi^{(J', \ell')}(v \cdot v') \gamma_5 + \xi^{(J', \ell')}(v \cdot v') \gamma_5' \right]. \] (8)

Inspection shows that the \( \gamma_5 \) and \( \gamma_5' \) terms are redundant, so that we may write, to this order,

\[
\langle D^{(J', \ell')}(v') | h^{(0)}_0 | B(v) \rangle = v_{\mu_1} \cdots v_{\mu_k} \xi^{(J', \ell')}(v \cdot v') \text{Tr} \left[ M^{(0)}_{\ell_1 \cdots \ell_k} (v') \right]. \] (9)

Thus, a single form factor is needed to this order, regardless of the spin of the final meson.

States with \( J' = 0^-, \ell = j + 1/2, \xi = 0 \) may be thought of as radial excitations of the ground states \( D^{(*)} \). Because of the heavy quark symmetry, and the orthogonality of these states with respect to the ground states which have the same quantum numbers, we must have

\[
\langle D^{(J', 0^-)}(v') | h^{(0)}_0 | B(v) \rangle = (v \cdot v' - 1) \xi^{(J', 0^-)}(v \cdot v') \text{Tr} \left[ M^{(0)}_{\ell_1 \cdots \ell_k} (v') \right]. \] (10)

where the superscripts \( (n) \) denote the nth radial excitation. That is, these amplitudes must vanish as \( v' \to v \). Note, too, that all of the other amplitudes \( (j \neq 1/2) \) vanish trivially at the non-recoil point.

We end this section by noting that we can use similar methods to enumerate the form factors that occur in the corresponding heavy to light transitions. For such a transition, say \( B \rightarrow K^{(*)} \), where \( K^{(*)} \) is some excited light meson, we write

\[
\langle K^{(*)}(p) | h^{(0)}_0 | D(v) \rangle = \text{Tr} \left[ R(\eta, p) \right]. \] (11)