The Quark Propagator in Quantum Chromodynamics: A Study in Landau Gauge

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Abstract. We examine the feasibility of dynamically broken chiral symmetry in QCD with zero-bare-mass quarks interacting via singular gluon exchange when the gluon propagator has the infrared behavior, $D_{\mu\nu} \sim q^{-4}$. An infrared analysis is carried out by employing a regulator prescription to study the dynamical content of the gap equation. Ward-Takahashi identities and the Dyson equations obeyed by the flavor vertex functions. The transverse parts of the latter are included. Besides the chiral symmetric solution, we obtain the solutions:

a) $S(p) = -M^{-1} = \text{constant}$

b) $S(p) \sim p(p^2)^{-1/2} + \text{constant}$, which is devoid of poles. Both solutions are indicative of quark confinement.

1. Introduction

The precise nature of the details of the dynamical breakdown of chiral symmetry $SU_f(N_F) \times SU_R(N_F)$ to $SU_f(N_F)$ is still not firmly established. The problem of the Nambu-Goldstone (NG) realization [1,2] of chiral symmetry is thus still an unresolved problem in Quantum Chromodynamics [3] (QCD). Yet, we have gained much insight into the problem as a result of the work of Cornwall [4], Ball and Zachariasen [5], Pagels [6], Kogut and Susskind and others [7]. More recently we had investigated solutions of Dyson equations of QCD which lead to the conclusion [8] that there may exist a causal connection between the NG realization of chiral symmetry and quark confinement.

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We believe that it is necessary to examine the equations of motion in a theory based on an assumed infrared behavior of the gluon propagator, which also employs a simple known form for the quark–antiquark Bethe–Salpeter scattering kernel. The basic procedure for such a study and the necessary approximations employed have been described in the pioneering work by Lane [9]. We need to make modifications as are appropriate.

The exact, full Bethe–Salpeter quark–antiquark scattering kernel that enters in the Dyson equations plays an important role in the analysis. Without an approximation for this kernel, the equations are intractable. The infrared behavior of QCD is determined by the infrared behavior of the kernel which is a sum of two distinct pieces: $K = K_p + K_{NP}$, where the perturbative piece $K_p$ admits the familiar skeleton graph expansion [10] and the part $K_{NP}$ denotes the nonperturbative instanton contributions [6]. When the forces are dominated by single gluon exchange, the kernel can then be represented by the Ladder graph approximation, $K_p \approx K_{L}$, which is the leading term of the skeleton graph series. That the kernel is well approximated by the ladder kernel in an asymptotically free theory was established by Lane for large momenta, where the instantons are unimportant. We shall assume without proof for small momenta, where $K_{NP}$ is expected to dominate over $K_p$, that the leading (most singular) term in $K$ has the ladder form with a singular effective gluon propagator, $D_{\mu\nu} \sim q^{-4}$. The work of Ball et al. [11] and West [12] lends strong support for this hypothesis. This approximation to the dynamics may be regarded as an effective strong coupling approximation since the running coupling constant $g(q^2) \sim (1/q^2)$ as $q^2 \to 0$. If chiral breaking
2. Dyson Equations and Ward-Takahashi Identities

Ward-Takahashi (WT) identities and the Dyson equations obeyed by the flavor vertex functions when approximated, of the gap equation together with the refinement, in the ladder effective 'strong coupling' ladder approximation. What gluon exchange forces represented by an effective ment. Ball et al. [11] have established that this singular nature of the propagator, (1.1) immediately produces infrared singular behavior, ~ q⁻⁴:

\[ D^{\mu\nu}(q) = \delta^{\mu\nu} \Delta^{\mu\nu}(q) = \delta^{\mu\nu} [g_{\mu\nu} q^2 / q^4 - g_{\mu\nu}] (\mu^2 / q^4) \]  (1.1)

where µ is a mass dimension characteristic of confinement. Ball et al. [11] have established that this singular behavior implies confinement in pure QCD. This ansatz thus, in a sense, implements or assumes confinement [16] and is equivalent to a running coupling constant \( g(q^2) \sim q^{-2} \) for small \( q \).

The procedure we have outlined is quite in the spirit of Lane's work which employs the Landau gauge, a running coupling constant in QCD and the singular nature of the gap equation together with the Ward–Takahashi (WT) identities and the Dyson equations obeyed by the flavor vertex functions when the dynamics of the theory is governed by the infrared singular gluon propagator. The singular nature of the gluon propagator, (1.1) immediately produces infrared divergent integrals in the various Dyson equations. In order to isolate and control this divergence, we shall introduce a regulator prescription. This will be seen to be different from the procedure used by Swift et al. [16] and the one introduced in Cornwall's work [4].

The paper is organized as follows. Section 2 introduces the Dyson equations. The derivation of the gap equation and the consequences of the vertex equations are briefly outlined here. Section 3 deals with the computations based on the infrared regulator prescription which lead to a gap equation that is tractable. The solutions are examined in Section 4. Some comments in conclusion are contained in Section 5.

2. Dyson Equations and Ward–Takahashi Identities

The framework of standard continuum QCD field theory upon which the infrared analysis will be developed has the following ingredients. The proper renormalized, color singlet, flavor non-singlet, vector and axial-vector current vertex functions obey the Ward–Takahashi (WT) identities

\[ q^a \Gamma^{a\mu}(p, p') = \frac{1}{2} \lambda^a S^{-1} (p') - S^{-1}(p) \frac{1}{2} \lambda^a, \]  (2.1)

\[ q^a \Gamma^{a}_{\mu\nu}(p, p') = Z_4 Z_2^{-1} [S^{-1}(p) \gamma_5 \frac{1}{2} \lambda^a + \gamma_5 \frac{1}{2} \lambda^a S^{-1}(p)], \]  (2.2)

where \( q = p' - p, \lambda^a \) is the flavor \( SU(N_F) \) matrix and \( a = 1, 2, \ldots, N_F^2 - 1 \). The renormalized quark propagator has the form

\[ S(p) = \hat{p} F(p^2) + G(p^2), \]  (2.3)

\[ S^{-1}(p) = A(p^2) \hat{p} - M(p^2), \]  (2.4)

which admits spontaneous breakdown of chiral symmetry signified by the statement

\[ \{ S^{-1}(p), \gamma_5 \} = -2 \gamma_5 M(p^2) \neq 0. \]  (2.5)

The form in (2.4) is consistent with the assumption that flavor \( SU(N_F) \) symmetry and color \( SU_c(3) \) symmetry are respected by the vacuum state:

\[ \{ \lambda^a, S(p) \} = [ \lambda^a, S(p) ] = 0, \]  (2.6)

where \( \lambda^a_c \) is the color \( SU(3) \) matrix. Equation (2.4) is then the most general invariant structure in the covariant Landau gauge [17]. The renormalized quark flavor non-singlet current vertex functions obey the Dyson equations

\[ \Gamma^{a\mu}_{\mu}(p, p') = \frac{1}{2} \lambda^a Z_4 \gamma_\mu \gamma_\mu - (2\pi)^{-4} \int d^4k S(k) \Gamma^{a\mu}_{\mu}(k, k') \]  (2.7)

\[ \Gamma^{a}_{\mu\nu}(p, p') = \frac{1}{2} \lambda^a Z_4 \gamma_\nu \gamma_\mu - (2\pi)^{-4} \int d^4k S(k) \Gamma^{a}_{\mu\nu}(k, k') \]  (2.8)

where \( q = p' - p = k' - k; K(p, q, k) \) is the full, renormalized quark–antiquark Bethe–Salpeter kernel. The renormalization constants obey the condition [9,18]

\[ Z_4 Z_2^{-1} < \infty. \]  (2.9)

Next we must deal with a solution to the WT identities, (1.1) and (2.2). A convenient solution specifying the longitudinal part, especially of the vector vertex, is quite well known [19]. An invariant form that is free of kinematical singularities [20] was employed in BZ, which may be rewritten more conveniently in terms of \( A \) and \( M \). The solution for the vector vertex is thus

\[ \Gamma^{a}_{\mu\nu}(p, p') = \frac{1}{2} \lambda^a \left\{ \frac{1}{2} (A + \bar{A}) \gamma_\mu + (M - \bar{M})(p^2 - p'^2)^{-1} \right. \]  (2.10)

\[ \left. \left( \gamma_\mu \hat{p} + \gamma_\mu \gamma_5 (M - \bar{M})(p^2 - p'^2)^{-1} \left[ 2 \gamma_5 \gamma_\mu \hat{p}' + (p^2 + p'^2) \gamma_\mu \right] + \Gamma^{a}_{\mu\nu} \right) \right\} \]

where \( \bar{A} = A(p^2), \bar{M} = M(p^2) \). The case of the axial-vector vertex, however, requires greater care because of the presence of the \( N_G \) pole. This would cause no problem if the transverse parts were neglected as was the case in [8]. Since the transverse part must have no kinematical singularities, it becomes necessary to employ an ansatz somewhat different from (2.10). Thus,

\[ \Gamma^{a}_{\mu\nu}(p, p') = \frac{1}{2} \lambda^a Z_4 Z_2^{-1} \left\{ \frac{1}{2} (A + \bar{A}) \gamma_\mu \gamma_5 \right. \]  (2.11)