Analysis of the wave configuration resulting from the termination of an inverse Mach reflection

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Abstract. An analytical model for solving the wave configuration which is formed when an inverse Mach reflection terminates after its triple point collides with the reflecting surface has been developed. The predictions of the model were compared with available experimental results and good agreement was obtained.

Key words: Shock wave reflections, Inverse Mach reflection

1. Introduction

Mach reflection is a three shock wave confluence which was discovered more than a century ago by Mach (1878). Its main structure consists of three shock waves, namely: the incident shock wave, the reflected shock wave the Mach stem and one slipstream. These four discontinuities meet at a single point known as the triple point, Courant and Friedrichs (1948) indicated that Mach reflection can be subdivided into three types depending upon the direction of propagation of the triple point as shown in Fig. 1. Figure 1a illustrates a Mach reflection in which the triple point moves away from the reflecting surface. This configuration was termed by Courant and Friedrichs (1948) as direct Mach reflection (D,MR). A stationary Mach reflection (S,MR) in which the triple point moves parallel to the reflecting surface is shown in Fig. 1b. The triple point of the Mach reflection shown in Fig. 1c. propagates towards the reflecting surface. This Mach reflection was termed by Courant and Friedrichs (1948) as inverted Mach reflection (I,MR), and was later retitled as inverse Mach reflection, (I,MR).

The nature of an I,MR which is a reflection typical of truly unsteady flows was described in detail by Takayama and Ben-Dor (1985). Since in the case of an I,MR the triple point moves towards the reflecting wedge surface, the I,MR is, by its nature a temporary reflection which terminates when its triple point collides with the reflecting surface. Once the I,MR has terminated, a new wave structure, whose configuration is the subject of this study, is formed.

The reflection process of an I,MR is shown schematically in Figs. 1c - 1e. A typical wave configuration of an I,MR is shown in Fig. 1c. Unlike a D,MR, for an I,MR the slipstream is directed away from the reflecting surface since the triple point T, propagates towards the reflecting surface as shown by the dashed line. The point along the reflecting surface where the I,MR terminates is the point where the triple point trajectory (dashed dotted line in Fig. 1c) intersects the reflecting surface. As shown in Fig. 1d, at this instant both the Mach stem m, and the slipstream s, vanish. At a later time a new wave configuration is formed. In this new wave configuration (shown in Fig. 1e), the incident shock wave i, reflects from the reflecting surface as a regular reflection RR. The reflected shock wave r, of this RR terminates at a new triple point T* which is connected to the reflecting surface by a shock wave n. It can be shown experimentally that this shock wave is not necessarily straight but is normal to the reflecting surface at its foot. The slipstream at this new triple point, s1 hits the reflecting surface and reflects from it as a new slipstream, s2. The wave configuration shown in Fig. 1e is maintained as long as the reflecting surface remains straight. Moreover it is self similar.

An I,MR occurs when a planar shock wave propagates towards a concave surface and reflects from it (Takayama and Ben-Dor 1983; Ben-Dor and Takayama 1986/7) or when a planar shock wave propagates towards a concave double wedge and reflects from it (Ben-Dor et al. 1987). Photographs of the wave configurations which are formed when the I,MR of these two examples termi-
2. Analytical Model

Figure 3a is an enlargement of the wave configuration shown in Fig. 1e which as mentioned previously is formed when an InMR terminates. The various flow regions are numbered from 0 to 4. The reflection point of the RR is P, the foot of the shock wave n, is at point Q and the slipstream reflects from the reflecting surface at point R.

If a frame of reference is attached to the reflection point P, the flow in region 0 flows in a direction parallel to the reflecting surface towards the incident shock wave i, with a velocity \( u_0 \) and an angle of incidence \( \phi_1 \) where \( u_0 \) and \( \phi_1 \) can be calculated from

\[
\phi_1 = 90^\circ - \theta_w \quad \text{and} \quad u_0 = V_s / \sin\phi_1 \tag{1}
\]

where \( \theta_w \) is the angle of the reflecting surface and \( V_s \) is the velocity of the incident shock wave, i.e. \( V_s = M_i a_0 \) where \( a_0 \) is the local speed of sound in region 0 and \( M_i \) is the incident shock wave Mach number. Upon passing through the incident shock wave, i, the flow is deflected by an angle \( \theta_1 \) and its velocity changes to \( u_1 \). This flow encounters the reflected shock wave, r, with an angle of incidence \( \phi_2 \). Upon passing through the reflected shock wave, r, it is deflected by an angle \( \theta_2 \) to become again parallel to the reflecting surface. Thus \( \theta_1 = \theta_2 \).

Thereafter, the flow in region 2 passes through the shock wave n and enters region 3 from which it cannot flow out since region 3 is bounded by the slipstream \( s_1 \) and the reflecting surface.

As mentioned earlier, the wave configuration shown in Fig. 3a is self-similar. Thus the entire structure grows linearly with time. Let us consider Fig. 3 where the wave configuration is drawn at two different times, say \( t \) and \( t + \Delta t \). Within the time interval \( \Delta t \) point Q moved to \( Q' \), point R to \( R' \) and the triple point \( T^* \) to \( T'^* \). Let us define the velocity of point Q with respect to point P as \( V_n \) and the velocity of the triple point \( T^* \) with respect to point P as \( dl/dt \) where \( l \) is the length of the